High-performance Randomized Matrix Computations for Big Data Analytics and Applications

Rank-k SVD

\[ A \approx U_k \Sigma_k V_k^T \]

where \( U_k \) is a large matrix, \( \Sigma_k \) is a diagonal matrix, and \( V_k \) is a matrix of eigenvectors. The columns of \( U_k, \Sigma_k, \) and \( V_k \) are the leading left singular vectors and right singular vectors of \( A \), respectively. The diagonal entries of \( \Sigma_k \) are the \( k \) largest singular values of \( A \).

Members
- Takahiro Katagiri (Nagoya U., Japan) : AT (ppOpen-AT), parallel eigenvalue algorithms, and supercomputer implementations.
- Weichung Wang (National Taiwan U., Taiwan): Numerical linear algebra, parallel computing, and AT (surrogate-assisted turning).
- Su-Yun Huang (Institute of Statistical Science, Academia Sinica, Taiwan) : Mathematical statistics and machine learning (random sketching algorithm).
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- Osni Marques (LBNL, USA) : Eigenproblem and its implementation (LAPACK, SVD algorithms).
- Feng-Nan Hwang (National Central U., Taiwan): Eigenproblem and its parallelization (SLEPc, SVD algorithms)
- Tosho Endo (TITECH, Japan) : System software (optimizations for hierarchical memory and adaptation of its AT)

Algorithm 1 Randomized SVD with single sketch (rSVD)

Require: Input \( A \) (real \( m \times n \) matrix), \( k \) (desired rank of approximate SVD), \( p \) (oversampling parameter), \( \ell = k + p \) (dimension of the sketched column space), \( q \) (power of projection).

Ensure: Approximate rank-\( k \) SVD of \( A \rightarrow U \_k, \Sigma \_k, V \_k \)

1. Sample a random matrix for low-dimensional projection.
2. Project \( A \) to the corresponding low-dimensional subspace.
3. Find a small-sized SVD in the low-dimensional subspace.

One Step Moving of iSVD

Algorithm 2 Integrated SVD with multiple sketches (iSVD).

Require: Input \( A \) (real \( m \times n \) matrix), \( k \) (desired rank of approximate SVD), \( p \) (oversampling parameter), \( \ell = k + p \) (dimension of the sketched column space), \( q \) (power of projection), \( N \) (number of random sketches).

Ensure: Approximate rank-\( k \) SVD of \( A \rightarrow U \_k, \Sigma \_k, V \_k \)

1. Sample \( q \) random matrices \( \Omega [i] \) for \( i = 1, \ldots, N \).
2. Assign \( Y [i] \leftarrow (A^T)^p \Omega [i] \) for \( i = 1, \ldots, N \) with \( \Omega [i] = \Omega \text{ (parallel)} \).
3. Compute \( Q [i] \) whose columns are orthogonal basis of \( Y [i] \) (in parallel).
4. Integrate \( Q \leftarrow \left\{ Q [i] \right\} _{i=1}^N \) by Algorithm 3 or Algorithm 4.
5. Compute SVD of \( \hat{Q} \rightarrow W, \hat{\Sigma}, V, \hat{\Sigma} \) (in parallel).
6. Assign \( \hat{U} \leftarrow W \hat{\Sigma} V^T \).
7. Extract the largest \( k \) singular-pairs from \( \hat{U} \), \( \hat{\Sigma} \), \( \hat{V} \) to obtain \( U \), \( \Sigma \), \( V \).

Kolmogorov-Nagumo Average

\[ Q + Q \_1 \leftarrow \varphi \left( \frac{1}{N} \sum_{i=1}^N \varphi (Q [i] Q [i]) \right) \]

\[ P \_1 \leftarrow \varphi \left( \frac{1}{N} \sum_{i=1}^N \varphi (P [i] P [i]) \right) \]

\[ Q + Q \_1 \leftarrow \varphi \left( \frac{1}{N} \sum_{i=1}^N \varphi (Q [i] Q [i]) \right) \]

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