Introduction

Chaotic attractor and Unstable periodic orbits within the chaotic attractor in the Lorenz system

- There are infinitely many unstable periodic orbits embedded with chaotic attractor.
- Chaos dances between the unstable periodic orbits.
- How does chaos dance between the unstable periodic orbits?
  
  Mathematical model based on network

Model

How to construct network of UPOs from chaos time series

1. Detect many UPOs on a Poincare section (UPOs as nodes in a network)
2. Calculate a chaos orbit on the Poincare section for an initial condition
3. Calculate Euclid distance between the all UPOs and chaos orbit on the Poincare section
4. Select the nearest UPO from the chaos (call the UPO *1*)
5. Calculate chaos on the Poincare section in going back to the section
6. Calculate Euclid distance between the all UPOs and chaos orbit on the Poincare section
7. Select the nearest UPO from the chaos (call the UPO *2*)
8. Provide a link from the UPO 1 to the UPO 2
9. Repeat (2)-(8)

Results

Lorenz system
\[
\frac{dx}{dt} = \sigma(y-x), \quad \frac{dy}{dt} = rx - y - xz, \quad \frac{dz}{dt} = xy - bz \quad (\sigma=10, b=8/3, r=28)
\]

UPOs which are used in this study

We detect almost complete UPOs with up to period 14

\#UPOs

\[\text{Saki 2007}\]

A part of constructed network

- Distribution of the out degree shows that “scale free network”.
- Same results are confirmed for the Hennon map.
- No clear correlation between period and out degree, and in- and out-degrees

Relationship between averaged quantities along chaos and UPOs

- Almost UPOs with high out degrees are UPOs whose average quantities agree with those of chaos

\[\text{The standard deviation decreases as } n \text{ increases} \rightarrow \text{almost UPOs with high out degree are “good”}\]

Summary

We have constructed mathematical model of chaos based on network of UPOs:

- Out degree
  - “Scale free network” (Same results for the Hennon map)
  - No clear correlation with period and in degree

- Out degree vs statistic quantities
  - Static quantities along UPOs with high out degree agree with those along chaos (Same results for more high-order static quantities, e.g. Lyapunov exponents and so on)