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Kengo Nakajima (Information Technology Center, The University of Tokyo, Japan)

Innovative Multigrid Methods

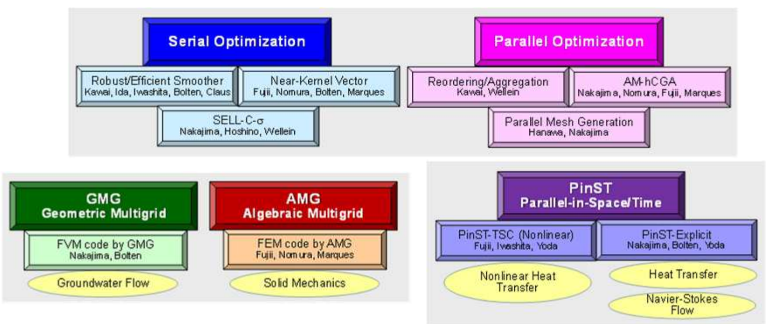
T. Iwashita[1], K. Nakajima (Leading-PI)[2,4], T. Hanawa[2], A. Ida[2], T. Hoshino[2], N. Nomura[2], A. Fujii[3], R. Yoda[3], M. Kawai[4], S. Ohshima[5], M. Bolten (Co-PI)[6], L. Claus[6], G. Wellein[7], O. Marques[8]
1: Hokkaido U., 2: U. Tokyo, 3: Kogakuin U., 4: RIKEN R-CCS, 5: Kyushu U., 6: U. Wuppertal, Germany, 7: FAU, Germany, 8: LBNL, USA

Background

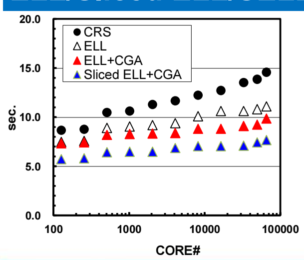
- Multigrid (MG)
  - ✓ Scalable multilevel method for solving linear equations
  - ✓ GMG (Geometrical Multigrid) and AMG (Algebraic)
  - ✓ Suitable for large-scale problems
  - ✓ Generally, number of iterations until convergence for multigrid method is kept constant as the problem size changes.
  - ✓ The parallel multigrid method is expected to be one of the most powerful tools on exa-scale systems.
  - ✓ Applied to rather well-conditioned problems (e.g. Poisson's equation)
  - ✓ Many sophisticated methods for robustness of MG have been developed for ill-conditioned problems derived from real-world scientific and engineering applications.
- Robust and efficient algorithms for GMG and AMG methods, and PinST towards the Exascale/Post-Moore Era.

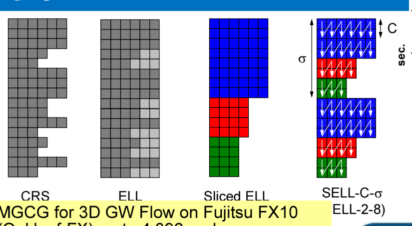
Development: 2-Year Project (FY.2018 & 2019)

- Serial Optimization for both of GMG & AMG
  - ✓ Robust & Efficient Smoother (MS-BMC-GS, Multiplicative-Schwartz type Block Multicolor Gauss-Seidel) (Kawai, Ida, Iwashita, Bolten, Claus)
  - ✓ Matrix Storage Format by SELL-C-σ (Nakajima, Hoshino, Wellein) [Kreutzer, Wellein et al. 2014]
  - ✓ Extraction of Near-Kernel Vectors (Fujii, Nomura, Bolten, Marques)
- Parallel Optimization for both of GMG & AMG
  - ✓ Parallel Reordering/Aggregation (Kawai, Wellein)
  - ✓ AM-hCGA (Adaptive Multilevel Hierarchical Coarse Grid Aggregation (hCGA)) (Nakajima, Nomura, Fujii, Marques)
  - ✓ Fast Parallel Mesh Generation by IME/Burst Buffer (Hanawa, Nakajima)
- Parallel in Space/Time (PinST)
  - ✓ PinST-TSC (Time Segment Correction) for Nonlinear Problems (Fujii, Iwashita, Yoda)
  - ✓ PinST-Explicit (Explicit Time Marching (Nakajima, Bolten, Yoda)
- Applications
  - ✓ GMG: 3D Groundwater Flow
  - ✓ AMG: 3D Solid Mechanics
  - ✓ PinST-TSC: Nonlinear Heat Transfer
  - ✓ PinST-Explicit: Heat Transfer, Incomp. Navier-Stokes with Pressure Correction
- Target Systems
  - ✓ Oakforest-PACS (U.Tokyo) : Intel Xeon Phi
  - ✓ ITO-A (Kyushu U.): Intel Xeon (Skylake)
  - ✓ New System (Hokkaido U.): Intel Xeon (Skylake)
- Open-Source Library
  - ✓ Various Types of Applications
  - ✓ One of the First Practical Library of MG including PinST, especially PinST-Exp



ELL/Sliced ELL/SELL-C-σ

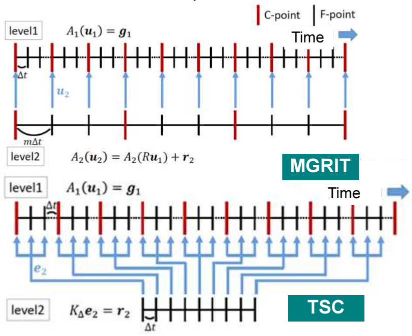


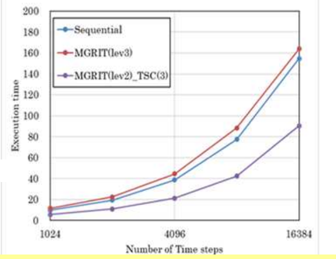


MGCG for 3D GW Flow on Fujitsu FX10 (Oakleaf-FX) up to 4,096 nodes, 17,179,869,184 DOF [Nakajima 2014]

Parallel-in-Space/Time (PinST)

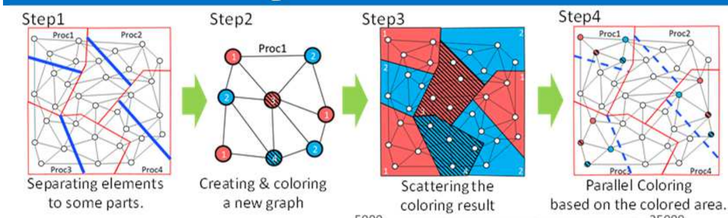
- Background
  - ✓ Domain-Decomposition: Parallel in Space Direction
  - ✓ New Approach with Parallel Computation in Time Direction (Parallel-in-Space/Time, PinST)
  - ✓ MGRIT (Multigrid-Reduction-in-Time) (R.D. Falgout et al., 2014)
- PinST/TSC
  - ✓ MGRIT was very successful in various types of applications including nonlinear ones, but it suffers from instability at coarse-level grid in time
  - ✓ TSC (Time Segment Correction)(Kaneko, Fujii, Iwashita et al., 2017) requires no coarse time step at coarser levels in time domain
- PinST/Exp
  - ✓ PinST has been mainly applied to implicit problems due to constraint of time step. But, it is really needed in explicit methods, where many time steps are needed.
  - ✓ New method for PinST with explicit time marching (PinST-Exp), where we apply coarser mesh in space at the coarser level in time

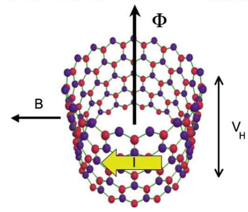




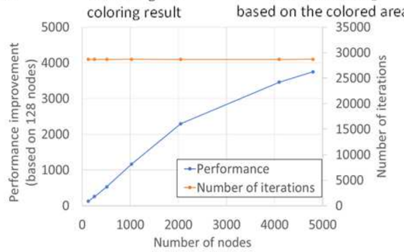
Efficiency of 1D Problem: MGRID+TSC is the best [Kaneko, Fujii, Iwashita et al. 2017]

Parallel Reordering





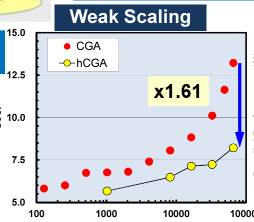
Source : The electronic properties of graphene



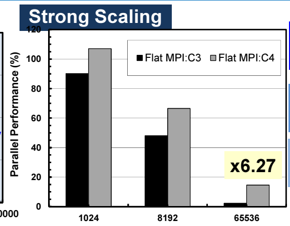
Effect of Hierarchical Reordering for Graphene Application with 500M DOF on 4,800 Nodes of the Fujitsu FX10 (Oakleaf-FX) using Blocked/Shifted ICCG, Constant Iterations for a wide Range of Node Number [Kawai et al. 2017]

AM-hCGA (Adaptive Multilevel hCGA)

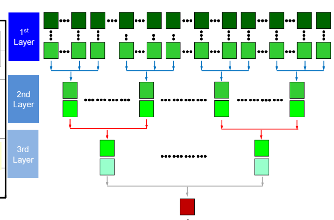
Weak Scaling



Strong Scaling



MGCG for 3D GW Flow on Fujitsu FX10 (Oakleaf-FX) up to 4,096 nodes, 17,179,869,184 DOF, Effects of hCGA [Nakajima 2014]



AM-hCGA (Adaptive Multi Level hCGA (Hierarchical Coarse Grid Aggregation)