機械学習に基づく流体変数の未来予測と数学的背景（jh190070-MDJ)

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中井 拳吾氏 (東京海洋大学) との共同研究

- K. Nakai and Y. Saiki (2018). Machine-learning inference of fluid variables from data using reservoir computing, Phys. Rev. E 98, 023111. https://arxiv.org/abs/1805.09917
- K. Nakai and Y. Saiki (2020). Machine-learning construction of a model for a macroscopic fluid variable using the delay-coordinate of a scalar observable, Discrete and Continuous Dynamical Systems S, online first. https://arxiv.org/pdf/1903.05770

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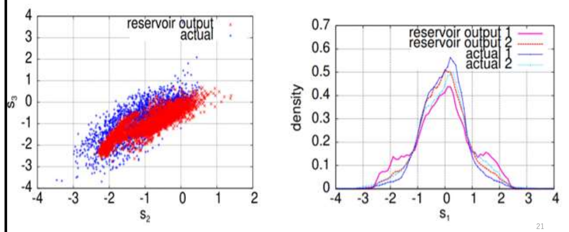
Poincaré section　　Distribution



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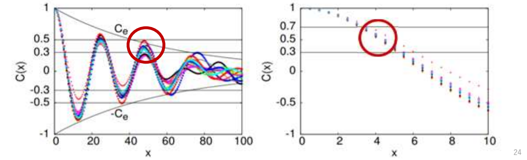
Auto-correlation function C(x) and its envelope Ce(x)

$$C(x)=\frac{\frac{1}{J}\sum_{j=0}^{J-1}(R_\lambda(t_0+j\Delta t^*)-\bar{R}_\lambda)(R_\lambda(t_0+j\Delta t^*+x)-\bar{R}_\lambda)}{\sqrt{\frac{1}{J}\sum_{j=0}^{J-1}(R_\lambda(t_0+j\Delta t^*)-\bar{R}_\lambda)^2}\sqrt{\frac{1}{J}\sum_{j=0}^{J-1}(R_\lambda(t_0+j\Delta t^*+x)-\bar{R}_\lambda)^2}}$$



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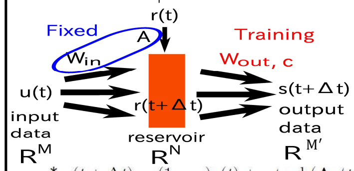
Reservoir computation



$M, M' \ll N$
$10^0 \sim 10^3$　$10^3 \sim 10^5$
(M = M′ for full-inference)

* $\mathbf{r}(t+\Delta t)=(1-\alpha)\mathbf{r}(t)+\alpha\tanh(\mathbf{A}\mathbf{r}(t)+\mathbf{W}_{\mathrm{in}}\mathbf{u}(t))$

$\mathbf{A}, \mathbf{W}_{\mathrm{in}}$: sparse random matrix, whose maximal eigenvalue is controlled
α: nonlinearity parameter ($\alpha=0.3$).

We determine　$\mathbf{W}_{\mathrm{out}}$ and c s.t.

$t^\forall < T$　　$\mathbf{W}_{\mathrm{out}}\mathbf{r}(t+\Delta t)+c\approx\mathbf{s}(t+\Delta t)$.

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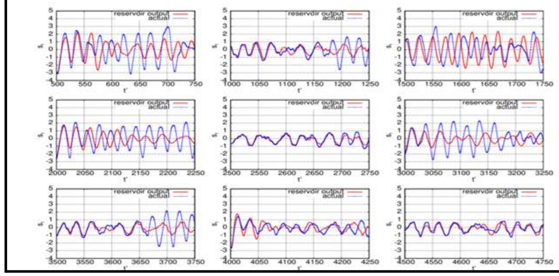
A single reservoir model can infer time-series from various initial conditions for some time



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Summary

- We infer time-series of both microscopic and macroscopic variables of fluid flow by machine-learning technique using reservoir computation without a prior knowledge of a physical process.
- In order to generate a time-series data of a macroscopic variable of a fluid flow, we do not need to go back to the microscopic dynamics.

We have especially succeeded in constructing a closed form equation of a fluid flow describing macroscopic behavior only from data.
Time-delay coordinate of an input vector should be chosen as follows:
- Delay-time Δτ is so that the auto-correlation function C is 0.45<C(Δτ)<0.55 for the first time.
- Dimension M is chosen so that the envelope Ce of C is 0.35<Ce(MΔτ)<0.40.

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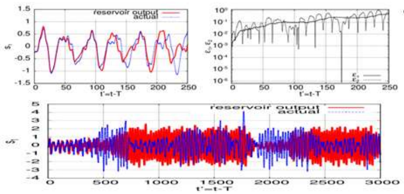
Modelling of the Reynolds number dynamics using the delay-coordinate　(Takens 1981, Sauer, Yorke and Catagli 1991)

$$\mathbf{u}(t)=(\tilde{R}_\lambda(t),\tilde{R}_\lambda(t-\Delta\tau),\cdots,\tilde{R}_\lambda(t-(M-1)\Delta\tau))^{\mathrm{T}}\quad \tilde{R}_\lambda(t):=\frac{\sqrt{(2/3)E(t)}\lambda}{\nu}=\sqrt{\frac{20E(t)^2}{3\nu\epsilon(t)}}$$
$$\mathbf{s}(t)=(\tilde{R}_\lambda(t),\tilde{R}_\lambda(t-\Delta\tau),\cdots,\tilde{R}_\lambda(t-(M-1)\Delta\tau))^{\mathrm{T}}$$

$$\epsilon(t)=2\nu\sum_{\kappa\in D}\sum_{\zeta=1}^{3}|\kappa|^2\left(\mathcal{F}_{[v_\zeta]}(\kappa,t)\right)^2,$$



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The number of successful cases for each set of the time-delay Δτ and the dimension M of the coordinate under two criteria

(i) the time average along $|\hat{s}_1(t')|<3$ for $t'\le 3000$,
(ii) the error $\varepsilon_2(t')=|s_1(t')-\hat{s}_1(t')|=|\tilde{R}_\lambda(t')-\hat{\tilde{R}}_\lambda(t')|<e_{60}$ for all $t'\le 60$,
(iii) the error $\varepsilon_2(t')<e_{90}$ for all $t'\le 90$,

(a) $(e_{60}, e_{90})=(0.14, 0.30)$

$\Delta\tau \setminus M$	10	11	12	13	14	15	16	17	18	19	20
3.0	0	0	0	0	0	1	19	24	43	37	27
3.5	0	0	0	11	20	28	57	48	21	11	7
4.0	0	3	18	43	107	59	21	14	2	4	5
4.5	3	14	43	54	21	15	8	1	1	1	0
5.0	10	24	26	19	9	1	1	1	0	0	0

(b) $(e_{60}, e_{90})=(0.13, 0.17)$

$\Delta\tau \setminus M$	10	11	12	13	14	15	16	17	18	19	20
3.0	0	0	0	0	0	0	3	6	10	8	4
3.5	0	0	0	2	3	5	6	4	1	3	1
4.0	0	0	2	8	14	10	1	4	1	0	1
4.5	1	1	8	14	1	0	1	0	0	0	0
5.0	2	4	6	6	3	0	1	0	0	0	0

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