学際大規模情報基盤共同利用・共同研究拠点(JHPCN)公募型共同研究 平成31年度採択課題
jh190003-NAJ

有限温度量子色力学のダイナミクス

鈴木　博
九州大学

Hiroshi Suzuki[1], Kazuyuki Kanaya[2], Yusuke Taniguchi[2], Shinji Ejiri[3], Takashi Umeda[4], Masakiyo Kitazawa[5] ([1]Kyushu U., [2]U. Tsukuba, [3]Niigata U., [4]Hiroshima U., [5]Osaka U.) [WHOT-QCD Collaboration]

To understand the state of matter at very high temperature and/or density, such as in the early universe, inside the neutron star, and in the heavy ion collision etc., it is crucial to know the Equation of State (EoS) of Quantum Chromo Dynamics (QCD), the fundamental theory of strong interactions. The ultimate goal of our project is to determine the EoS in the continuum by numerical simulations on the basis of lattice QCD with the Wilson-type quark action. For this, we are employing the energy–momentum tensor (EMT) and other physical quantities defined by the gradient flow (GF) and the Small Flow-$t$ime eXpansion (SFtX) method. We summarize achievements we made so far by utilizing the present and past JHPCN (and other) computational resources.

The Gradient Flow (GF) (Narayanan–Neuberger ('06), Lüscher ('09–)) is deformation of the gluon field $A_\mu(x)$ along a fictitious time $t \geq 0$ according to a gauge-invariant diffusion-type equation

$$\frac{\partial}{\partial t} B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \qquad B_\mu(t=0,x) = A_\mu(x),$$

where

$$G_{\mu\nu}(t,x) = \partial_\mu B_\nu(t,x) - \partial_\nu B_\mu(t,x) + [B_\mu(t,x), B_\nu(t,x)]$$

and

$$D_\nu G_{\nu\mu}(t,x) = \partial_\nu G_{\nu\mu}(t,x) + [B_\nu(t,x), G_{\nu\mu}(t,x)].$$

This deformation $A_\mu(x) \to B_\mu(t,x)$ may be regarded as a diffusive smearing of $A_\mu(x)$ over the region $|x| \sim \sqrt{8t}$. Remarkably, it has been shown that any local product of the flowed gauge field $B_\mu(t,x)$ with $t > 0$ automatically becomes a renormalized finite field (Lüscher–Weisz ('11)). This property of GF can be used to construct a universal expression for EMT [H.S., PTEP 2013, 083B03 ('13)].

The idea is that one can construct an operator of flowed fields which coincides with EMT with dimensional regularization at $t \to 0$ (the small flow-time limit); the latter satisfies Ward–Takahashi identities associated with the translational invariance and the former is independent of regularization adopted. In this way, one can obtain a universal expression of EMT that is usable even in lattice QCD. In QCD without quark fields (the quenched QCD) for which numerical simulations are relatively less expensive, it has been confirmed that this SFtX method works quite well as shown in Asakawa–Hatsuda–Itou–Kitazawa–H.S. PRD90, 011501 ('14), Kitazawa–Iritani–Asakawa–Hatsuda–H.S., PRD94, 114512 ('16), Iritani–Kitazawa–H.S.–Takaura, PTEP 2019, 023B02 ('19).

We apply this SFtX method to the $2+1$-flavor QCD. The coefficients required to this computation were worked out to the one-loop order in Makino–H.S., PTEP 2014, 063B02 ('14) and to the two-loop order in Harlander–Kluth–Lange, Eur. Phys. J. C78, 944 ('18). This method enable us to determine EoS without additional inputs such as the lattice beta function whose precise computation is quite demanding. Similar idea can be applied to other physical quantities such as fermion bi-linear operators [Hieda–H.S., Mod. Phys. Lett. A31, 1650214 ('16)] and the topological density, etc.

In this poster, we show the results for the $N_f = 2+1$ QCD, but with ud quarks somewhat heavier than nature ($m_\pi/m_\rho = 0.63$) and s quark of the physical mass [WHOT-QCD, PRD96, 014509 ('17); D95, 054502 ('17), arXiv:2005.00251 [hep-lat]]. In this study, we use gauge field configurations generated by non-perturbatively $O(a)$-improved Wilson quark action and the RG improved Iwasaki gauge action. The lattice spacing is fixed to $a0.07\,\mathrm{fm}$. $T = 0$ configurations are CP-PACS+JLQCD configuration on a $28^3 \times 56$ lattice. $T > 0$ configurations are WHOT-QCD configurations ($32^3 \times N_t$, $N_t = 4, 6, \ldots, 16$).

EoS obtained by SFtX method [arXiv:2005.00251 [hep-lat]]:

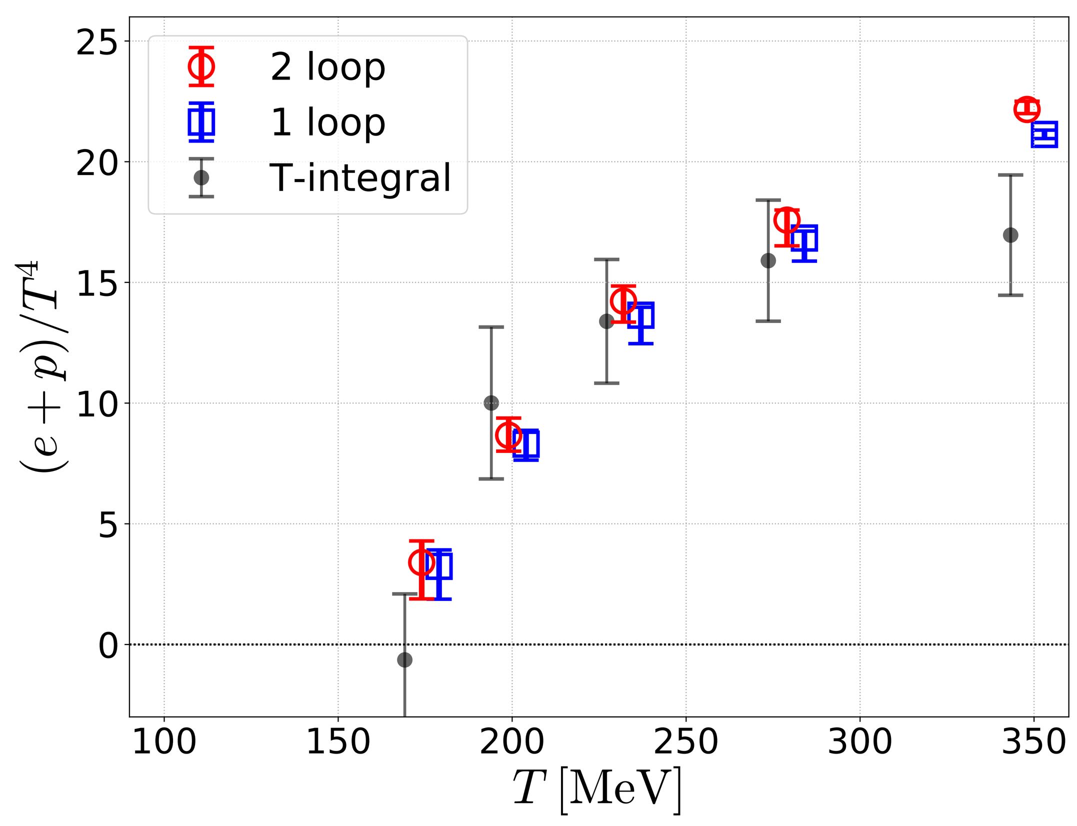

Figure: Entropy density $(\epsilon + p)/T^4$ as function of temperature T

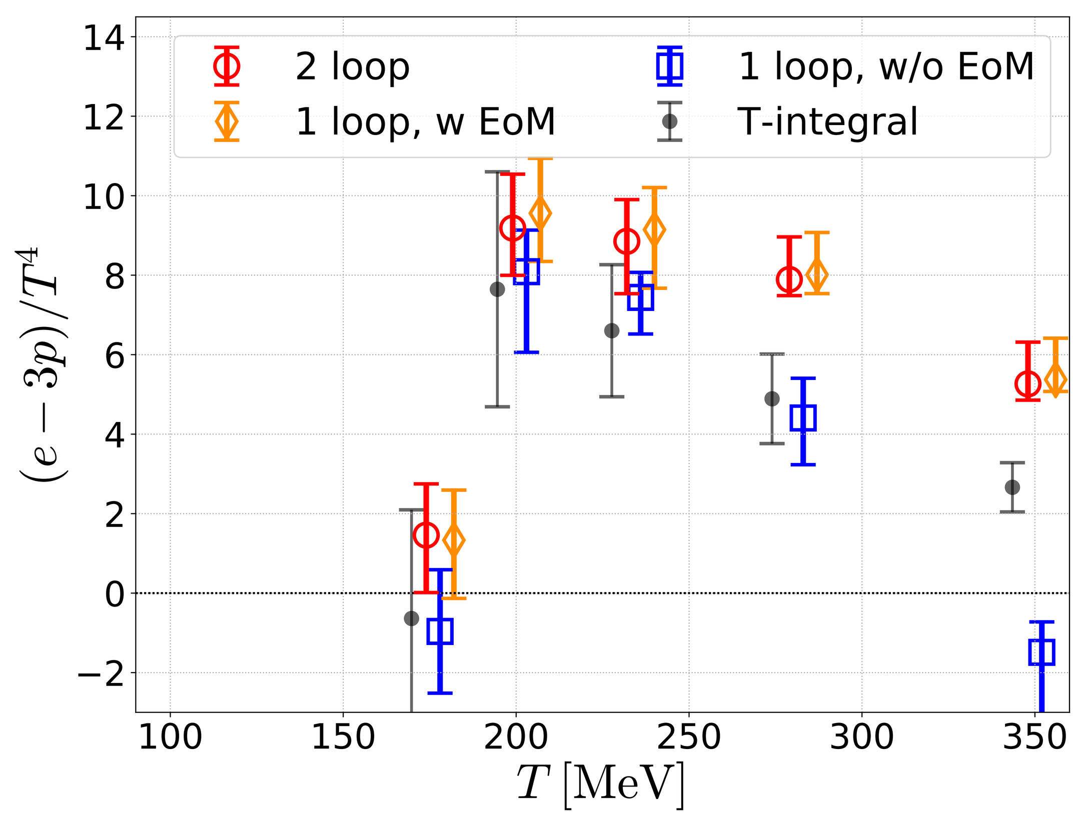

Figure: Trace anomaly $(\epsilon - 3p)/T^4$ as function of temperature T

For $T \leq 300\,\mathrm{MeV}$ ($N_t \geq 10$) with which the discretization error $O((aT)^2 = 1/N_t^2)$ is expected to be small, the entropy density obtained by the SFtX method is consistent with the result of a conventional method with much smaller statistical/systematic errors. On the other hand, it can be seen that the trace anomaly suffers from a large discretization error $T \leq 250\,\mathrm{MeV}$ ($N_t \geq 8$) associated with the Equation of Motion (EoM). This point should be further studied.

Chiral condensate/disconnected chiral susceptibility
[arXiv:2005.00251 [hep-lat]]:

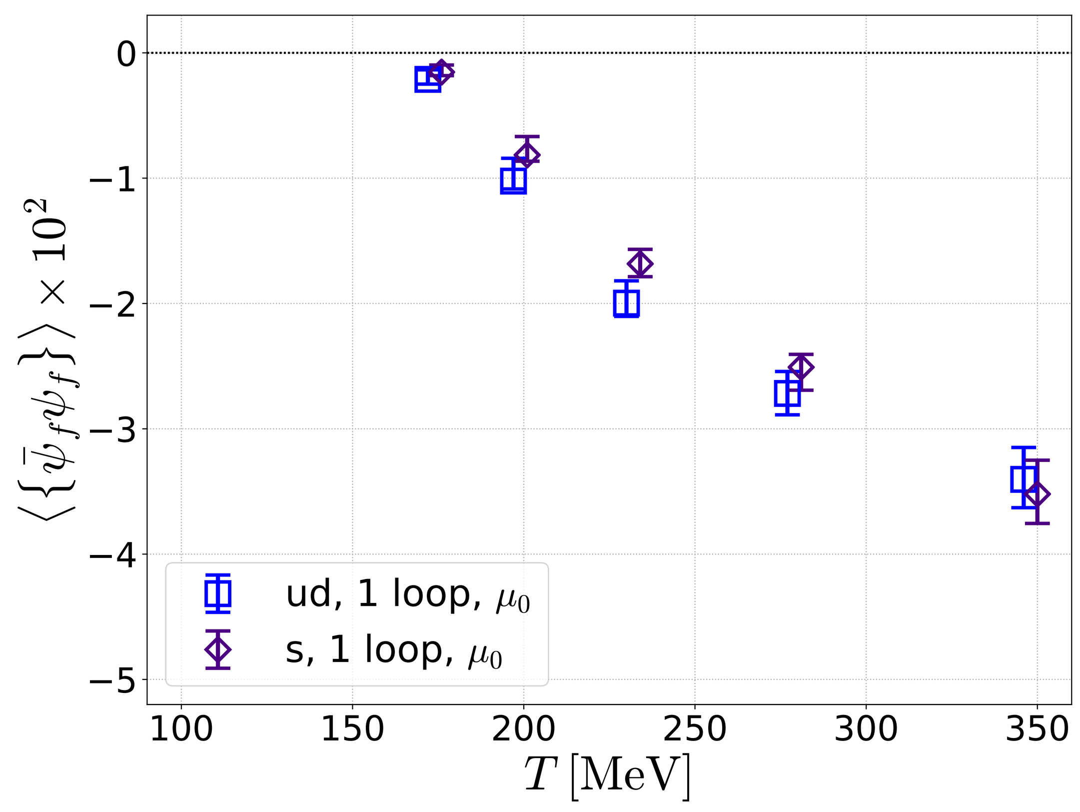

Figure: VEV subtracted chiral condensate $\langle\{\bar\psi_f\psi_f\}\rangle$ in the $\overline{\text{MS}}$-scheme at $\mu = 2$ GeV in GeV^3

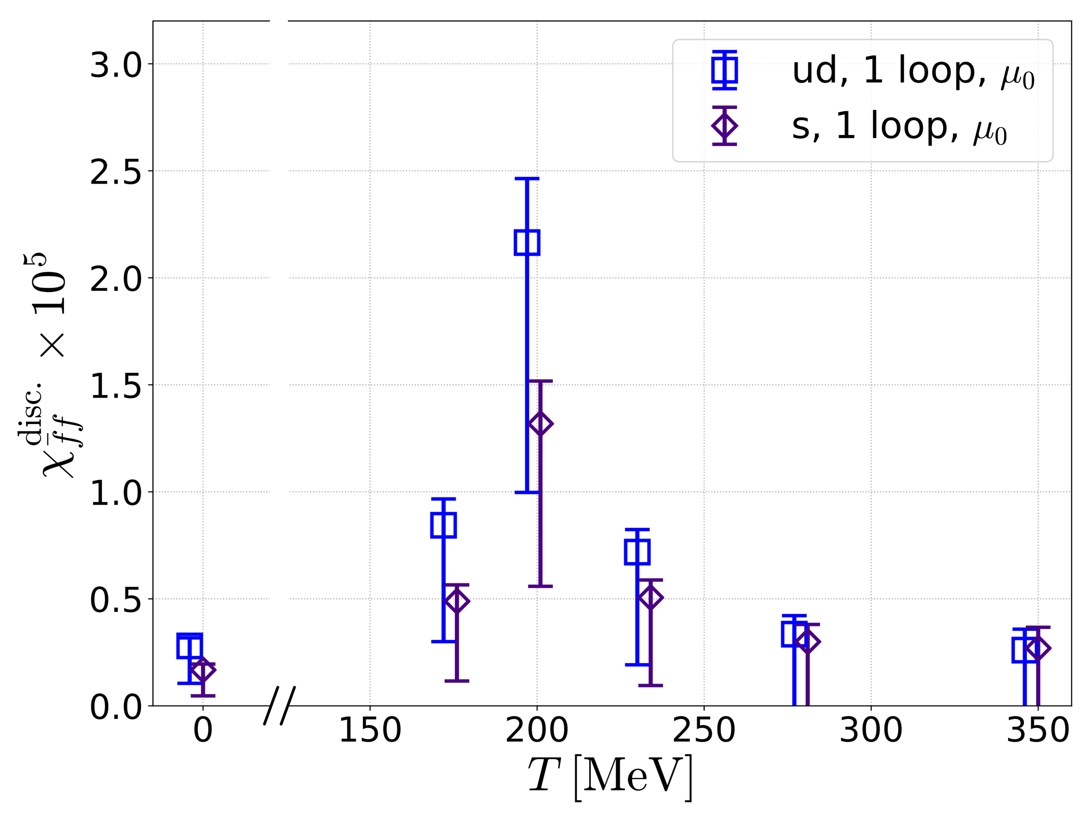

Figure: Disconnected chiral susceptibility $\chi^{\text{disc.}}_{\bar ff}$ in the $\overline{\text{MS}}$-scheme at $\mu = 2$ GeV in GeV^6

For the disconnect chiral susceptibility, we see a clear peak at $T \simeq 199$ MeV, which indicates the pseudo-critical point. We also note that the height of the peak looks increasing as we decrease the quark mass from s quark to ud quarks.

Topological susceptibility [Taniguchi–Kanaya–H.S.–Umeda, PRD 95, 054502 ('17)]:

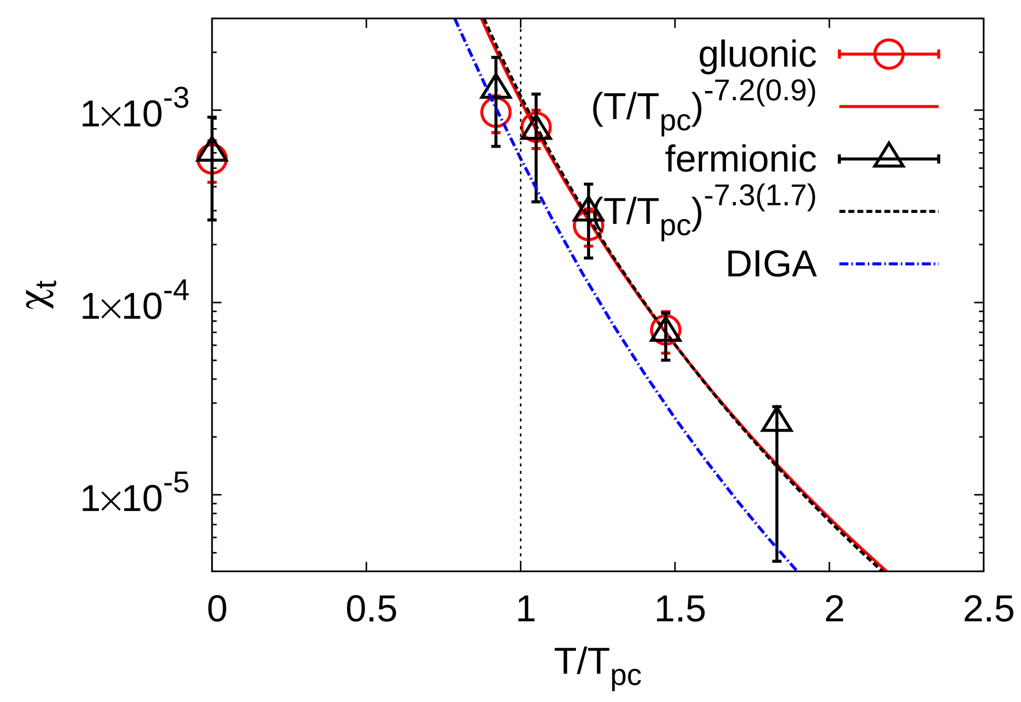

Figure: Topological susceptibility as a function of temperature in GeV^4

Gluonic and fermionic definitions of the topological susceptibility χ_t agree to each other as must be the case in the continuum limit. Power-low behavior is consistent with the dilute instanton gas approximation (DIGA) which predicts the exponent $\propto T^{-8}$.