[jh230028] Construction of a machine learning model of fluid flow with fluctuation of unstable dimensions

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1 Introduction.

Reservoir computing, a brain-inspired machine-learning technique that employs a data-driven dynamical system, is effective in predicting time series and frequency spectra in chaotic behaviors, including fluid flow and global atmospheric dynamics [1, 2, 4, 5, 6].

The extent to which a data-driven model using reservoir computing can capture the dynamical properties of original systems should be determined. Lu *et al.* [1] reported that a data-driven model has an attractor similar to that of the original system under an appropriate choice of parameters. Nakai and Saiki [3] confirmed that a single data-driven model could infer the time series of chaotic fluid flow from various initial conditions. They suggested that a data-driven model could reconstruct the attractor of the original dynamical system.

In our project, we analyze a data-driven model constructed using reservoir computing from a dynamical system point of view. In particular, we focus on reconstructing fluctuations of unstable dimensions in time development. High-dimensional chaotic dynamics are believed to have heterochaos, namely, the coexistence of dense sets of periodic orbits of different unstable dimensions. The property is one of the sources of structural instability, and it is interesting to investigate whether the property can be reconstructed by data-driven modeling. Data-driven models we analyze are those of the 96-Lorenz system and the macroscopic fluid flow.

2 Reservoir computation.

What's Reservoir computation?

- a relatively high-dimentional fixed neural-network composed of simple nonlinear dynamical systems
- determination of output layer
- For Lorenz system and Kuramoto–Sivashinsky system, inference [1, 5, 6]

2.1 Procedure of training.

○ 1st step (making a reservoir vector)

Making a reservoir vector $\mathbf{r}(t)$ correspond to decomposition of the input data $\mathbf{u}$ by using nonlinear function tanh:

$\mathbf{r}(t+\Delta t) = (1-\alpha)\mathbf{r}(t) + \alpha \tanh(\mathbf{A}\mathbf{r}(t) + \mathbf{W}_{\rm in}\mathbf{u}(t))$.

$\mathbf{A}, \mathbf{W}_{\rm in}$: sparse random matrix, whose maximal eigenvalue is controlled.

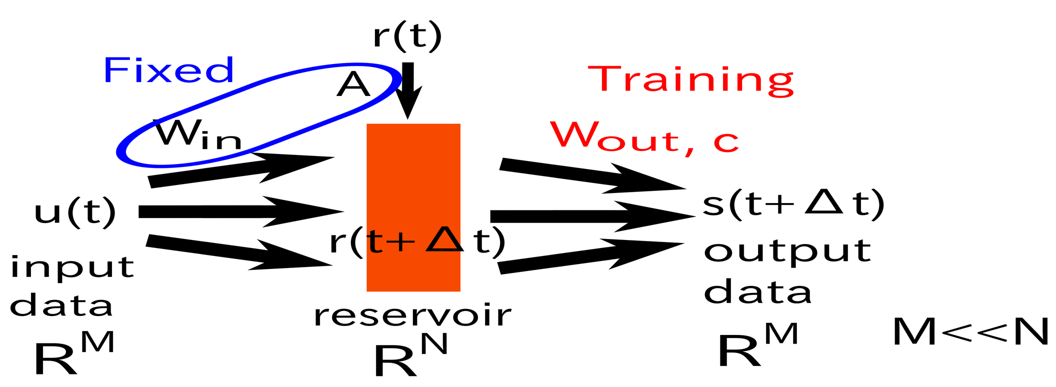

Figure 1: Schematic picture of a reservoir computing (training phase)

○ 2nd step (determination of output layer)

We determine $\mathbf{W}_{\rm out}$ s.t.

$$t^{\forall} < T \qquad \mathbf{W}_{\rm out}\mathbf{r}(t+\Delta t) \approx \mathbf{s}(t+\Delta t).$$

⇒ Determine them s.t. the form is minimized:

$$\sum_{l=1}^{L} \|(\mathbf{W}_{\rm out}\mathbf{r}(l\Delta t)) - \mathbf{s}(l\Delta t)\|^2 + \beta[Tr(\mathbf{W}_{\rm out}\mathbf{W}_{\rm out}^T)].$$

2.2 Procedure of inference.

Using the $\mathbf{W}^*_{\rm out}$, we infer the time-series $\mathbf{s}$.

$\hat{\mathbf{s}}(t) = \mathbf{W}^*_{\rm out}\mathbf{r}(t)$

$\mathbf{r}(t+\Delta t) = (1-\alpha)\mathbf{r}(t) + \alpha \tanh(\mathbf{A}\mathbf{r}(t) + \mathbf{W}_{\rm in}\hat{\mathbf{s}}(t))$.

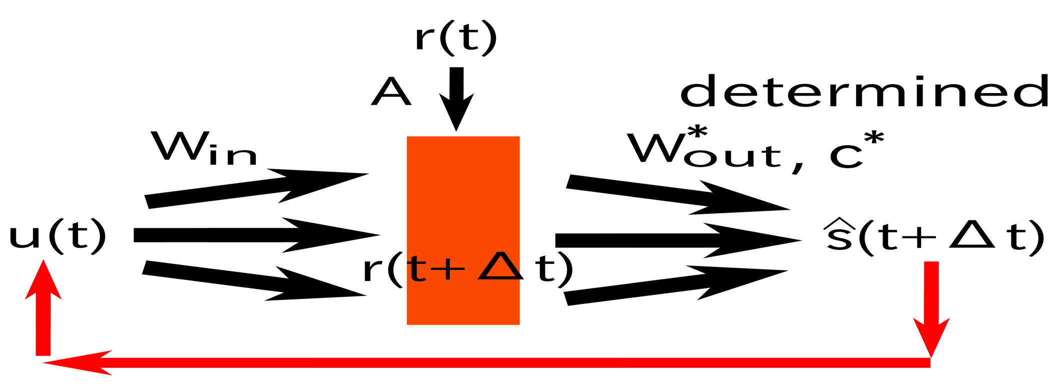

Figure 2: Schematic picture of a reservoir computing (prediction phase)

This reservoir system corresponds to the data-driven model of u.

3 Our project.

The first example is the data-driven model of the Lorenz [7], a dissipative N-dimensional ODE which is a model of some oscillating scalar atmospheric quantity described as

$$\frac{dx_k}{dt} = x_{k-1}(x_{k+1} - x_{k-2}) - x_k + f, \text{ for } k = 1, \ldots, N, \quad (1)$$

where the system has cyclical symmetry so $x_{N+k} = x_k$ for all $k = 1, \ldots, N$, and where f is a forcing parameter.

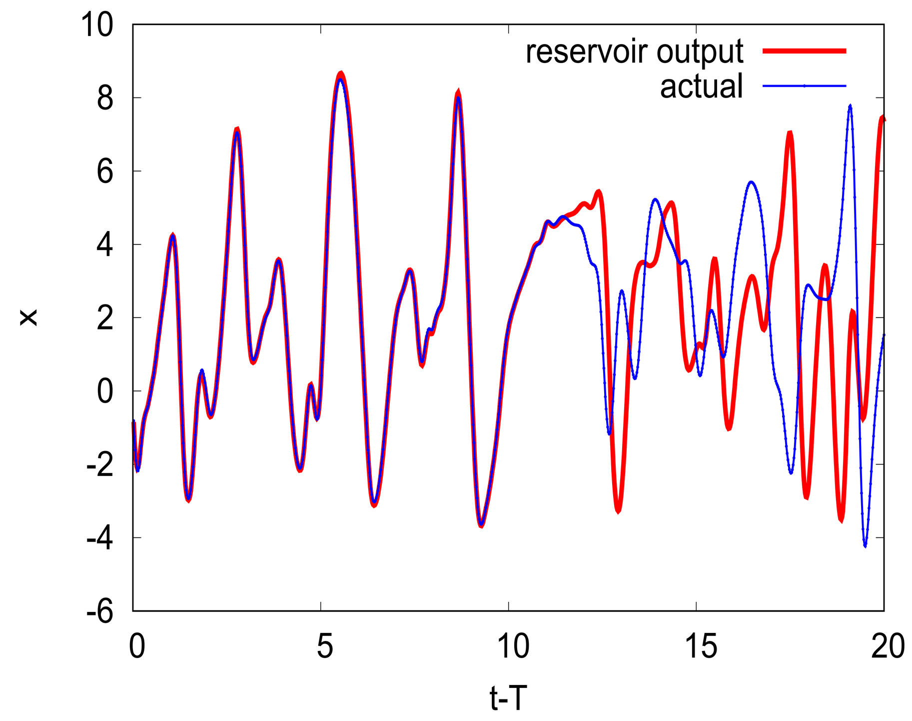

Figure 3: Short time inference of a variable of the Lorenz-96 system

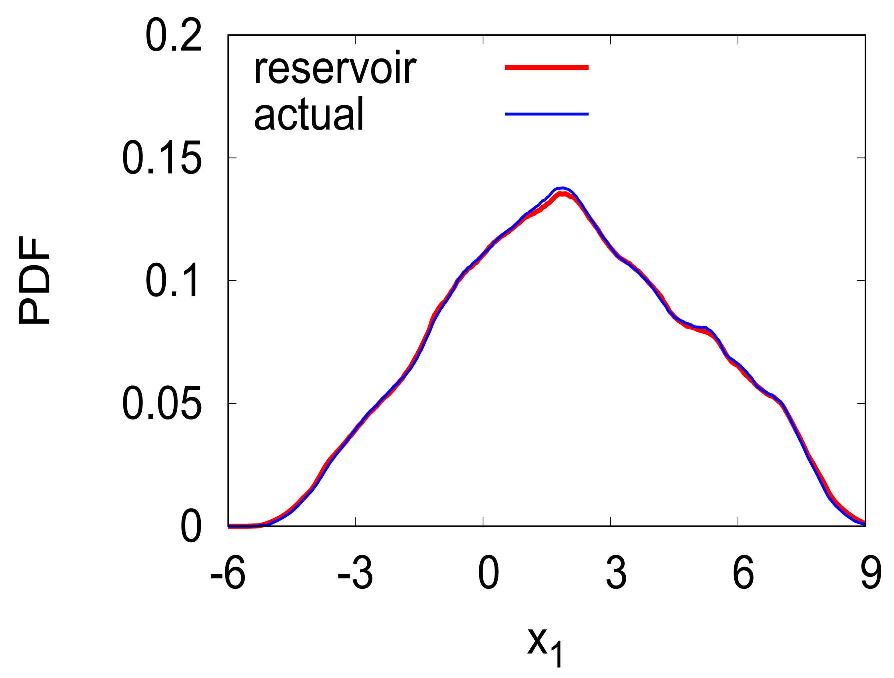

Figure 4: Density distribution of a variable.

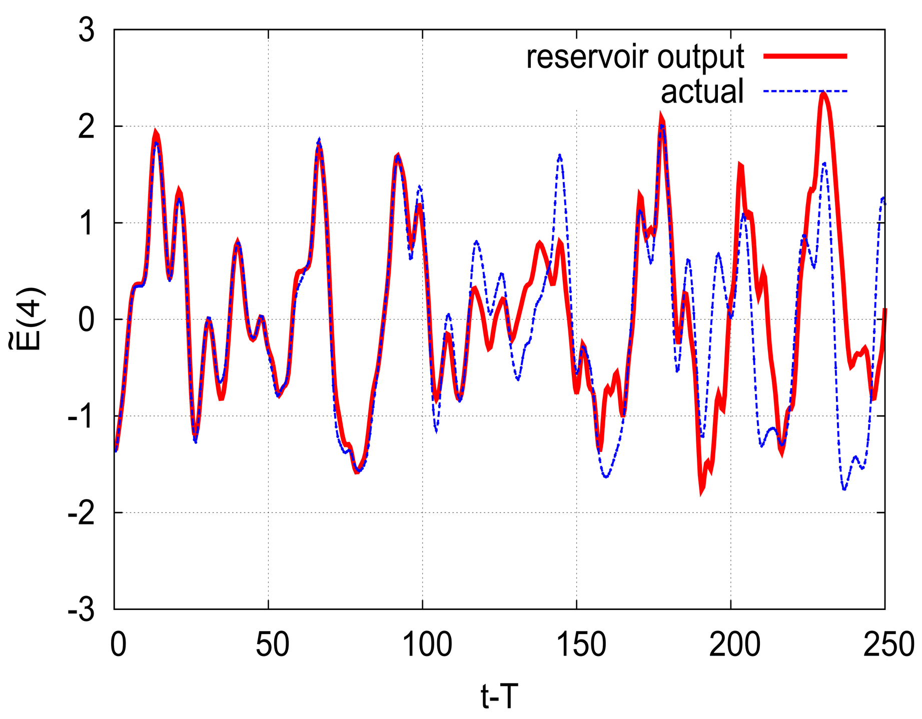

Figure 5: Short time inference of a variable of the fluid flow

Reference

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