Dirac流モノポールによるQCDのカラー閉じ込め機構のモンテ・カルロ研究

T. Suzuki, RCNP, Osaka Univ.
with K.Ishiguro, Kochi Univ., July 7, 2022

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1. Introduction

Color confinement problem in QCD not yet solved.

Almost half a century history !!!

1963: Quark model (Gell-Mann and Zweig): fractionally charged quarks ($Q_u = 2/3$, $Q_s = -1/3$) are proposed, but not observed upto now.

(Idea)

Quarks are assumed to have a color degree of freedom (RED, GREEN, BLUE) in addition to flavor degree of freedom and interact via 8 colored gauge fields $A_\mu^a(x) \quad (a = 1 \sim 8)$. **(Quantum Chromodynamics)**

Colors must be confined due to some unknown mechanism.

1974-75: Idea of dual superconductor (electric $\leftrightarrow$ magnetic) as the color-confinement mechanism ('tHooft-Mandelstam): Something color magnetic must be condensed.

$$L_{QCD} = -1/2\,\mathrm{Tr}(G_{\mu\nu})^2,$$
$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

$A_\mu(s)$ has only color-electric charge. How to define color magnetic quantity from $A_\mu(s)$? This is very difficult and quite different from the case in the usual superconductor theory.

The key point is to find a color magnetic quantity, a magnetic monopole in QCD from $A_\mu(s)$ alone without any artificial additional assumption like (partial) gauge fixing.

$$L = -\frac{1}{4}(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c)^2$$

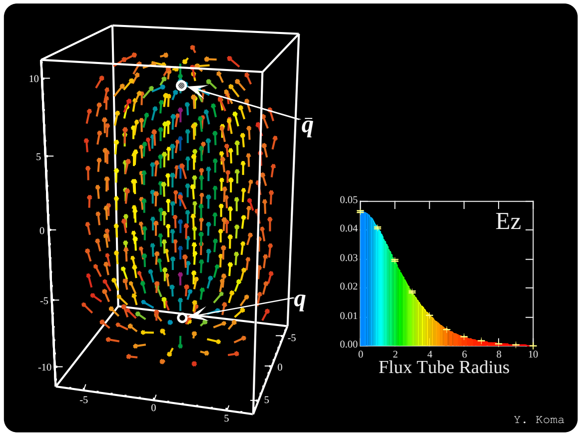


Figure 1: electric flux

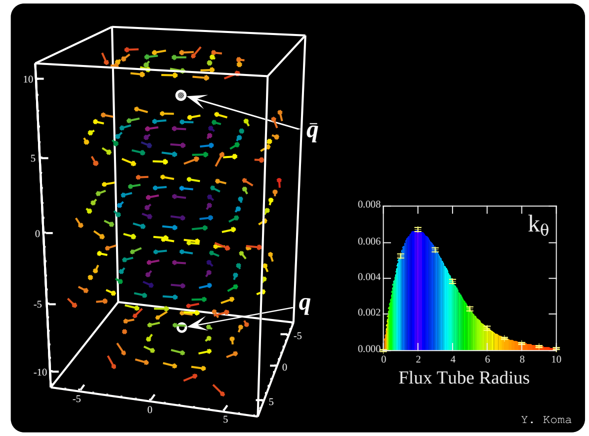
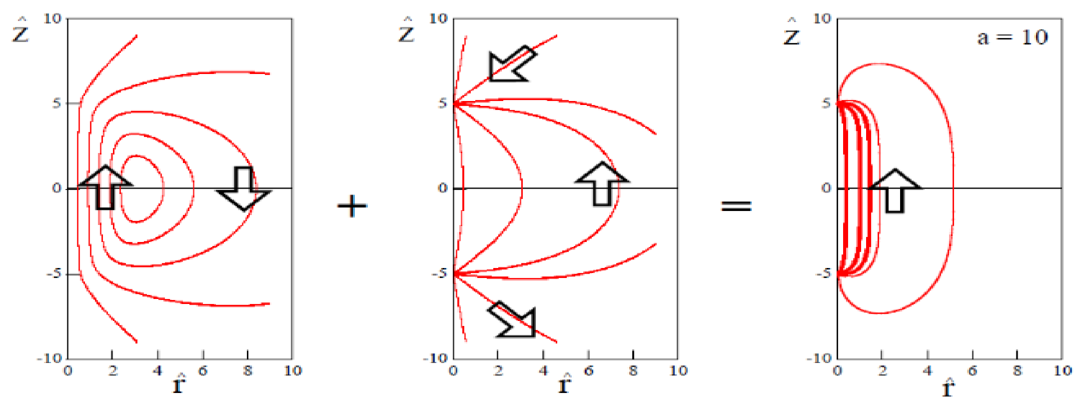


Figure 2: solenidal magnetic current

- **Profile of electric field in r-z plane**



(a) (b) (c)

Sum of (a) solenoidal and (b) Coulombic electric fields creates (c) a flux tube.

How to derive (probablly) Abelian conserved magnetic currents squeezing non-Abelian color electric flux starting only from $A^a_\mu(x)$ having color electric charges?

2.The purpose of 2022 JHPCN project

The author has found in 2014 that there are Abelian monopoles of the Dirac type in QCD. It is important to study if the Abelian dual Meissner effect can explain color confinement.In QCD, numerical Monte-Carlo calculations of QCD in the framework of finite-volume lattice with non-zero lattice spacing and studies of the continuum limit after taking the infinite-volume limit and the zero lattice spacing limit is the uniquie rigorous method.

Up to now, in SU2 QCD, the authors have obtained promizing results already.

In SU3 QCD, Abelian and monopole sominances as well as the Abelian dual Meissner effect have been studied without any additional assumptions smoothing the vacuum. But the continuum limit has not been studied yet, since in SU3, at present it is impossible to go to larger lattices for various coupling constants without introduing any additional assumptions smoothing the vacuum.

The main purpose of this 2022 JHPCN project is to prove the existence of the continuum limit of the monopoles of the Dirac type in SU3. Additionally, to study the effect of light dynamical quarks on Abelian monopoles is another purpose.

3. A complete new idea of magnetic monopoles in QCD

Note the Jacobi identities:

$$\epsilon_{\mu\nu\rho\sigma}[D_\nu, [D_\rho, D_\sigma]] = 0,$$

where $D_\mu \equiv \partial_\mu - igA_\mu$. Calculate explicitly:

$$\begin{aligned} [D_\rho, D_\sigma] &= [\partial_\rho - igA_\rho, \partial_\sigma - igA_\sigma] \\ &= -ig(\partial_\rho A_\sigma - \partial_\sigma A_\rho - ig[A_\rho, A_\sigma]) + [\partial_\rho, \partial_\sigma] \\ &= -igG_{\rho\sigma} + [\partial_\rho, \partial_\sigma] \end{aligned}$$

$[\partial_\rho, \partial_\sigma]$ can not be neglected in general!!

$D_\nu G^*_{\mu\nu} = 0 \rightarrow$ Non-Abelian Bianchi identity (NABI):

$f_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)\sigma^a/2$

$\partial_\nu f^*_{\mu\nu} = 0 \rightarrow$ Abelian-like Bianchi identity:

Jacobi identity + $[D_\nu, G_{\rho\sigma}] = D_\nu G_{\rho\sigma}$

$$
\begin{aligned}
\Longrightarrow D_\nu G^*_{\mu\nu} &= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} D_\nu G_{\rho\sigma} = -\frac{i}{2g}\epsilon_{\mu\nu\rho\sigma}[D_\nu, [\partial_\rho, \partial_\sigma]] \\
&= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}[\partial_\rho, \partial_\sigma] A_\nu = \partial_\nu f^*_{\mu\nu} \\
J_\mu = \frac{1}{2} J^a_\mu \sigma^a = D_\nu G^*_{\mu\nu} &= \partial_\nu f^*_{\mu\nu} = \frac{1}{2} k^a_\mu \sigma^a = k_\mu
\end{aligned}
$$

$k^a_\mu \neq 0 \rightarrow$ (color magnetic) Abelian-like monopole of the Dirac type:
$J^a_\mu \neq 0 \rightarrow$ Violation of NABI, $\partial_\mu k_\mu = \partial_\mu J_\mu = 0$

Color magnetic monopoles= Violation of non-Abelian Bianchi identity (VNABI)

$$[\partial_\rho, \partial_\sigma] A_\nu \neq 0 \Longrightarrow$$

Line singularities existing in gauge fields $A_\mu(x)$ themselves proposed by Dirac (1931) are the origin of the QCD monopoles and $N^2 - 1$ monopoles exist in $SU(N)$.

4. Lattice studies without any additional assumptions

Lattice Abelian fields and monopoles

Maximize $R = Re\,\mathrm{Tr}\, e^{i\theta_1(s,\mu)\lambda_1}U^\dagger(s,\mu) \implies$

$$\begin{aligned}
\theta_1(s,\mu) &= \tan^{-1}\frac{Im(U_{12}(s,\mu)+U_{21}(s,\mu))}{Re(U_{11}(s,\mu)+U_{22}(s,\mu))} \\
\theta_1(s,\mu\nu) &= \partial_\mu\theta_1(s,\nu)-\partial_\nu\theta_1(s,\mu) \\
&= \bar{\theta}_1(s,\mu\nu)+2\pi n_1(s,\mu\nu) \quad (|\bar{\theta}_1(s,\mu\nu)|<\pi)
\end{aligned}$$

$$\Downarrow$$

$$k_\mu^1(s) = \frac{1}{2\pi}\partial_\nu\bar{\theta}_1(s,\mu\nu)$$

Lattice monopole defined above following DeGrand-Toussaint is not gauge-invariant without additional gauge-fixing. But Elitzer's theorem says that gauge-invariant contents, if exist, can be extracted by Monte-Carlo average of gauge-variant quantities.

S. Elitzur, P.R. D12 (1975) 3978.

If the Abelian dual Meissner is the essence of the color confinement, it is expected $\sigma_F = \sigma_a = \sigma_m$. This is called as perfect Abelian and perfect monopole dominance with respect to the string tension. It is also possible to measure directly the Abelian dual Meissner effect.

Let us review the results obtained upto now.

(1) Pefect Abelian and monopole dominances with respect to the string tension

Evaluate

$$V_{\rm mon}(R) = -\frac{1}{aN_t}\ln\langle P_{\rm mon}(0)P^*_{\rm mon}(R)\rangle \ .$$

$$P_{\rm A} = \exp[i\sum_{k=0}^{N_t-1}\theta_1(s+k\hat{4},4)] = P_{\rm ph}\cdot P_{\rm mon}\ ,$$

$$P_{\rm ph} = \exp\{-i\sum_{k=0}^{N_t-1}\sum_{s'}D(s+k\hat{4}-s')\partial'_\nu\bar{\theta}_1(s',\nu 4)\}\ ,$$

$$P_{\rm mon} = \exp\{-2\pi i\sum_{k=0}^{N_t-1}\sum_{s'}D(s+k\hat{4}-s')\partial'_\nu n_1(s',\nu 4)\}$$

1. Perfect Abelian dominance (using Luscher's multilevel method)

Table 1: Simulation parameters: $N_{\rm sub}$ is the sublattice size divided and $N_{\rm iup}$ is the number of internal updates in the multilevel method.

β	$N_s^3 \times N_t$	$a(\beta)$ [fm]	$N_{\rm conf}$	$N_{\rm sub}$	$N_{\rm iup}$
5.60	$16^3 \times 16$	0.2235	6	2	10000000
5.70	$12^3 \times 12$	0.17016	6	2	5000000
5.80	$12^3 \times 12$	0.13642	6	3	5000000

Table 2: The string tension σa^2, the Coulombic coefficient c, and the constant μa.

$\beta = 5.6, 16^3 \times 12$	σa^2	c	μa
$V_{\rm NA}$	0.239(2)	-0.39(4)	0.79(2)
$V_{\rm A}$	0.25(2)	-0.3(1)	2.6(1)
$\beta = 5.7, 12^3 \times 12$			
$V_{\rm NA}$	0.159(3)	-0.272(8)	0.79(1)
$V_{\rm A}$	0.145(9)	-0.32(2)	2.64(3)
$\beta = 5.8, 12^3 \times 12$			
$V_{\rm NA}$	0.101(3)	-0.28(1)	0.82(1)
$V_{\rm A}$	0.102(9)	-0.27(2)	2.60(3)

2. Perfect monopole dominance

Table 3: Simulation parameters for the measurement of the static potential from $P_{\rm A}$, $P_{\rm ph}$ and $P_{\rm mon}$ in $SU(2)$ and $SU(3)$. $N_{\rm RGT}$ is the number of random gauge transformations.

	β	$N_s^3 \times N_t$	$a(\beta)$ [fm]	$N_{\rm conf}$	$N_{\rm RGT}$
$SU(3)$	5.60	$24^3 \times 4$	0.2235	910000	400
$SU(2)$	2.43	$24^3 \times 8$	0.1029(4)	7000	4000

Table 4: Best fitted values of the string tension σa^2, the Coulombic coefficient c, and the constant μa for the potentials $V_{\rm NA}$, $V_{\rm A}$, $V_{\rm mon}$ and $V_{\rm ph}$.

$SU(3)$ $(24^3 \times 4)$					
	σa^2	c	μa	FR(R/a)	$\chi^2/N_{\rm df}$
$V_{\rm NA}$	**0.178(1)**	0.86(4)	0.99(1)	5 - 9	1.23
$V_{\rm A}$	**0.16(3)**	0.9(11)	2.5(3)	5 - 9	1.03
$V_{\rm mon}$	**0.17(2)**		2.9(1)	4 - 7	1.08
$V_{\rm ph}$	$-0.0007(1)$	0.046(3)	0.945(1)	3 - 10	7.22e-08
$SU(2)$ $(24^3 \times 8)$					
$V_{\rm NA}$	0.0415(9)	0.47(2)	0.46(8)	4.1 - 7.8	0.99
$V_{\rm A}$	0.041(2)	0.47(6)	1.10(3)	4.5 - 8.5	1.00
$V_{\rm mon}$	0.043(3)	0.37(4)	1.39(2)	2.1 - 7.5	0.99
$V_{\rm ph}$	$-6.0(3) \times 10^{-5}$	0.0059(3)	0.46649(6)	7.7 - 11.5	1.02

We tried also $40^3 \times 6(\beta 5.75)$, $40^3 \times 8(\beta = 5.9)$ and $40^3 \times 10(\beta = 6.05)$ to study scaling, but unsuccessful.

Table 5: Simulation parameters for the measurement of the static potential and the force from $P_{\rm A}$, $P_{\rm ph}$ and $P_{\rm mon}$. $N_{\rm RGT}$ is the number of random gauge transformations.

β	$N_s^3 \times N_t$	$a(\beta)$ [fm]	$N_{\rm conf}$	$N_{\rm RGT}$
$SU2$, 2.20	$24^3 \times 4$	0.211(7)	6000	1000
2.35	$24^3 \times 6$	0.137(9)	4000	2000
2.35	$36^3 \times 6$	0.137(9)	5000	1000
2.43	$24^3 \times 8$	0.1029(4)	7000	4000

Table 6: Best fitted values of the string tension σa^2, the Coulombic coefficient c, and the constant μa for the potentials $V_{\rm NA}$, $V_{\rm A}$, $V_{\rm mon}$ and $V_{\rm ph}$.

$SU(2)$					
$24^3 \times 4$	σa^2	c	μa	FR(R/a)	$\chi^2/N_{\rm df}$
$V_{\rm NA}$	0.181(8)	0.25(15)	0.54(7)	3.9 - 8.5	1.00
$V_{\rm A}$	0.183(8)	0.20(15)	0.98(7)	3.9 - 8.2	1.00
$V_{\rm mon}$	0.183(6)	0.25(11)	1.31(5)	3.9 - 6.7	0.98
$V_{\rm ph}$	$-2(1) \times 10^{-4}$	0.010(1)	0.48(1)	4.9 - 9.4	1.02
$36^3 \times 6$	σa^2	c	μa	FR(R/a)	$\chi^2/N_{\rm df}$
$V_{\rm NA}$	0.072(3)	0.48(9)	0.53(3)	4.6 - 12.1	1.03
$V_{\rm A}$	0.073(2)	0.47(6)	1.10(2)	4.3 - 11.2	1.03
$V_{\rm mon}$	0.073(3)	0.46(7)	1.43(3)	4.0 - 11.8	1.01
$V_{\rm ph}$	$-1.0(1) \times 10^{-4}$	0.0132(1)	0.4770(2)	6.4 - 11.5	1.03
$24^3 \times 8$	σa^2	c	μa	FR(R/a)	$\chi^2/N_{\rm df}$
$V_{\rm NA}$	0.0415(9)	0.47(2)	0.46(8)	4.1 - 7.8	0.99
$V_{\rm A}$	0.041(2)	0.47(6)	1.10(3)	4.5 - 8.5	1.00
$V_{\rm mon}$	0.043(3)	0.37(4)	1.39(2)	2.1 - 7.5	0.99
$V_{\rm ph}$	$-6.0(3) \times 10^{-5}$	0.0059(3)	0.46649(6)	7.7 - 11.5	1.02

3. The Abelian dual Meissner effect

Table 7: Simulation parameters for the measurement of E_i^a and k^2 (Left). $\nabla \times \vec{E}$, $\partial_4 \vec{B}$, k_i^a (Right). id is the distance between two Polyakov loops. Nconf, Nran and Ns are numbers of configurations, random gauge copies and smearing, respectively.

E_i^a			
id	Nconf	Nran	Ns
d=3	20000	100	90
d=4	20000	100	90
d=5	80000	100	120
d=6	80000	100	120
k^2			
id	Nconf	Nran	Ns
d=3	80000	0	90
d=4	160000	0	90
d=5	960000	0	120
d=6	960000	0	120

the dual Ampère's law			
id	Nconf	Nran	Ns
d=3	20000	100	90
k_ϕ^a			
d=3	11200	3000	90
k_r^a			
d=3	9600	3000	90
k_z^a			
d=3	9600	3000	90

$$\rho_{conn}(O(r)) = \frac{\left\langle \mathrm{Tr}(P(0)LO(r)L^\dagger)\mathrm{Tr}P^\dagger(d)\right\rangle}{\langle \mathrm{Tr}P(0)\mathrm{Tr}P^\dagger(d)\rangle} - \frac{1}{3}\frac{\left\langle \mathrm{Tr}P(0)\mathrm{Tr}P^\dagger(d)\mathrm{Tr}O(r)\right\rangle}{\langle \mathrm{Tr}P(0)\mathrm{Tr}P^\dagger(d)\rangle},$$

Figure 3: Cylindrical coordinate (Left). The Abelian color electric field around static quarks for $d = 5$ at $\beta = 5.6$ on $24^3 \times 4$ lattices. (Right)

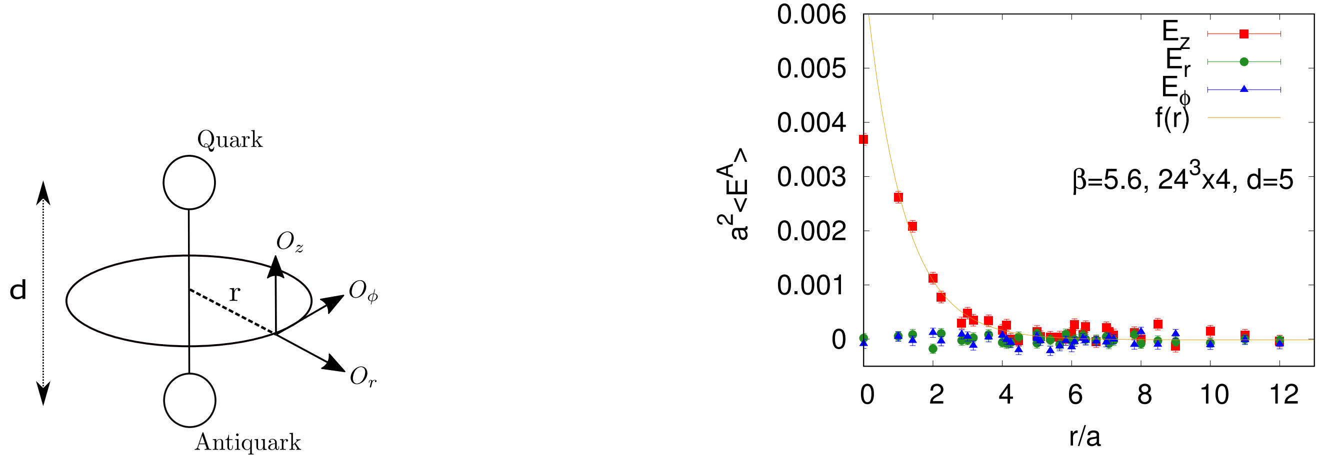
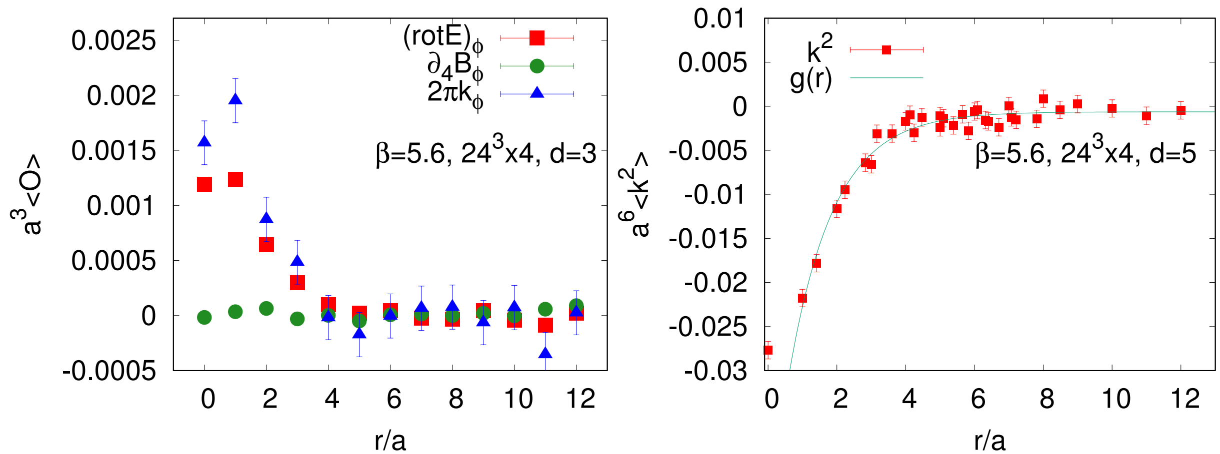


Figure 4: The dual Ampère's law with $d = 3$ at $\beta = 5.6$ on $24^3 \times 4$ lattices (Left). The squared monopole density with $d = 5$ at $\beta = 5.6$ on $24^3 \times 4$ lattices (Center). The Ginzburg-Landau parameter $\kappa = \lambda/\xi$ (Right).



d	$\sqrt{2}\kappa$
3	0.87(5)
4	0.93(7)
5	0.83(9)
6	0.9(2)

5. Study of the continuum limit in SU3

At present, only preliminary results are obtained.

(1) Abelian and monopole dominances 1) Wilson action on 32^4 lattice for $\beta = 5,8,\ 6.0,\ 6.2,\ 6.4$

2) Number of vacuum configurations=320

3) Non-Abelian data are cited from S. Necco and R. Sommer, Nucl. Phys. **B622**, 328 (2002)

Table 8: Abelian, monopole and photon string tensions

β	5.8	6.0	6.2	6.4
σ_a/σ	1.03(1)	1.123(2)	1.13(3)	1.40(1)
σ_m/σ	0.87(1)	0.827(7)	1.100(7)	1.01(3)
σ_p/σ	0.0062(1)			

(2) Blockspin study of Abelian monopoles and the continuum limit in $SU(3)$

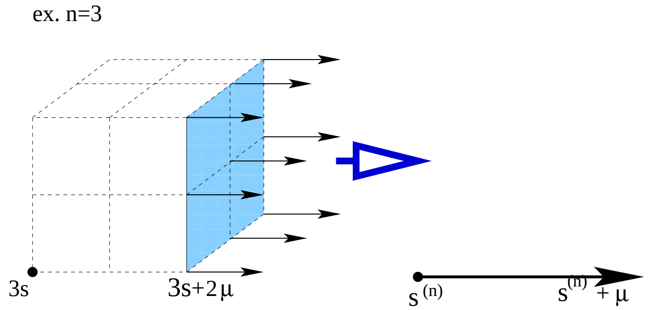
The lattice vacuum is contaminated with large amount of lattice artifact monopoles. To reduce lattice artifacts, gauge-fixing smoothing the vacuum are introduced.

1. Iwasaki action:
48^4 at $\beta = 2.3 \sim 3.5$:

2. Introduction of smooth gauge-fixings
Maximal Abelian and $U(1)^2$ Landau gauge (MAGU12): Maximization of
$R = \sum_{s,\mu}(U_\mu^\dagger(s)\vec{H}U_\mu(s)\vec{H})$ $\vec{H} = (\lambda_3, \lambda_8)$
We also studies another smooth gauge called Maximal Center gauge (MCG) in which $R = \sum_{s,\mu} |TrU_\mu(s)|^2$ is maximized.

3. Block-spin transformation s of monopoles.

Figure 5: Blockspin definition of monopoles:
T.L. Ivanenko et al., Phys. Lett. **B252**, (1990) 631



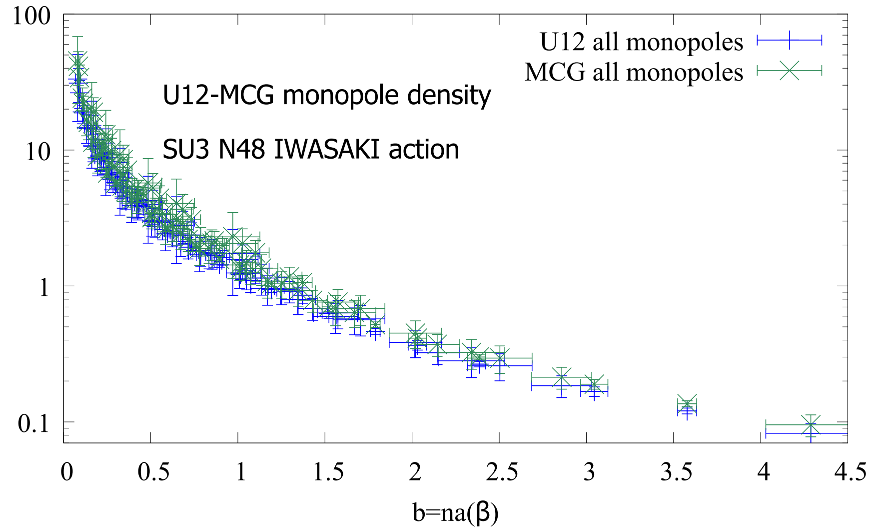
Monopole is defined on a a^3 cube and the n-blocked monopole is defined on a cube with a lattice spacing $b = na$.

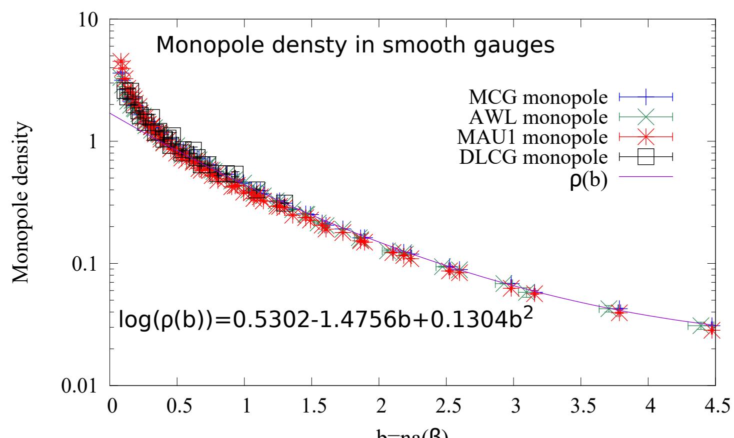
$$k_{\mu}^{(n)}(s_n) = \sum_{i,j,l=0}^{n-1} k_{\mu}(ns_n + (n-1)\hat{\mu} + i\hat{\nu} + j\hat{\rho} + l\hat{\sigma})$$

Evaluate a gauge-invariant density of the n-blocked monopole:

$$\rho = \frac{\sum_{\mu,s_n}\sqrt{\sum_a (k_\mu^{(n)a}(s_n))^2}}{4\sqrt{3}V_n b^3}$$

Figure 6: Comparison of the VNABI (Abelian-like monopoles) densities versus $b = na(\beta)$ in $SU3$ MAGU12 case (Up). For comparison, $SU(2)$ data are also shown (Down).





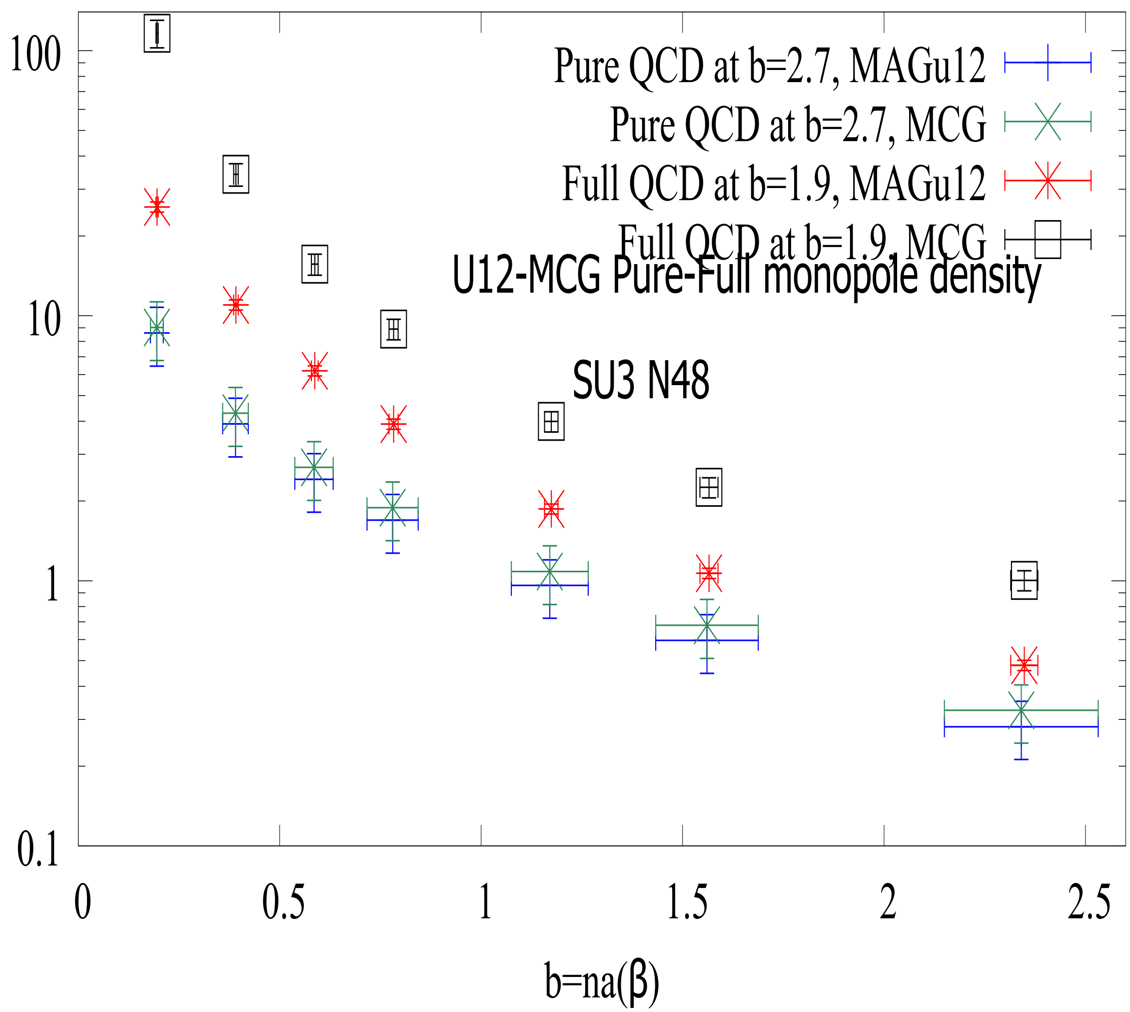
Summary

1. Clear scaling behaviors are observed up to the 12-step blockspin transformations for all β adopted. The density $\rho(a(\beta), n)$ is a function of $b = na(\beta)$ alone, i.e. $\rho(b)$. $n \to \infty$ means $a(\beta) \to 0$ for fixed $b = na$. Existence of the continuum limit!

2. When the vacuum becomes smooth enough shown here in MCG, DLCG, AWL, MAU1 of the SU(2) case, the same $\rho(b)$ is obtained. In SU(3) also, similar data are obtained in MAGU12 and MCG gauges.

3. The study in full QCD having light dynamical quarks is being studies.

Preliminary! Comparison between pure Iwasaki at $\beta = 2.7$ and PACS-CS data at $\beta = 1.9,\ m_\pi = 157 MeV$ on 48^4 having similar lattice spacings:



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