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# Innovative Multigrid Methods II

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## Purpose and Background

Robust and efficient algorithms for Multigrid(MG) methods towards the Exascale/Post-Moore Era.

- Multigrid (MG) for linear equations
  - ✓ Scalable multilevel method for solving linear equations
  - ✓ GMG (Geometrical Multigrid) and AMG (Algebraic)
  - ✓ The parallel multigrid method is expected to be one of the most powerful tools on exa-scale systems.
  - ✓ Many sophisticated methods for efficiency of MG have been developed for ill-conditioned problems derived from real-world scientific and engineering applications.
- MG for parallel in Space and Time
  - ✓ Parallel computation in time direction of simulation
  - ✓ It adds new dimension to ordinary parallelization of scientific simulation calculation

## Development: 3-Year Project (FY.2020 to 2022)

- GMG and AMG:
  - ✓ Research on smoothers

    Kawai, Ida, Nakajima, Bolten, Wellein, Alappat, Schreiber

    - ➢ Multicolor-block GS smoother for AMG is the original smoother developed by Our project members.
    - ➢ SpMV based smoothers such as Chebyshev smoother are known. We will also consider to accelerate these simple smoothers by fast sparse matrix data structures
  - ✓ Lower precision utilization for performance

    Nakajima, Hoshino, Wellein, Alappat, Schreiber, Ohshima

    - ➢ We will study on how to use the lower precision calculations for efficient multigrid solvers.
    - ➢ First, we will investigate various lower precision usage for solvers such as only coarse level calculation or lower precision Krylov preconditioning.
    - ➢ After we checked effective lower precision usage patterns, then we will study selection methods according to the problem matrix.
  - ✓ Acceleration technique with file IO optimization and communication

    Hanawa, Ohshima, Fujii, Nakajima, Yoda

    - ➢ Parallel linear solvers and mesh generation routines often need to read and write distributed matrix files. We will investigate efficient usage of the burst buffer functionality of supercomputers for mesh generation or sparse linear solvers.
    - ➢ As for halo communication, we will study fast Halo communication with user-level direct RDMA on Tofu, that will be available in Fugaku, post-Kei computer.
  - ✓ Evaluation with weak scaling and large sized problems

    Nakajima, Fujii, Marques, Ida, Tanaka

    - ➢ Our project already has spent several years to implement multigrid solvers such as GMG and AMG. We will analyze the performance of the solvers, and check the space for improvement.

- New approaches for PinST

  Ono, Iizuka, Fujii, Bolten, Iwashita, Takahashi, Speck, Schreiber, Yoda, Chen, Caklovic, Miyagi

  - ✓ Coarse level solver in Parareal method Iizuka and Ono studied the convergence of the Parareal method for hyperbolic PDEs focusing on coarse solver.
  - ✓ Convergence analysis on MGRIT preconditioning for linear problems
  - ✓ Time Segmented Correction (TSC) method and its enhancement, which is a new parallel time integration method for non-linear problems.

## Plan for FY.2021

This project is a second year of the 3-year project. The current state of the research will be presented as the research results of the last year. Here, we list up our plans on FY.2021.
As a second year, we will enhance the researches including solver evaluation on Fugaku.

- GMG
  - ✓ Researches on sparse matrix data structure like SELL-C-σ
  - ✓ Lower precision GMG solver
  - ✓ Specific application optimization including pGW3D-FVM
  - ✓ File IO optimization in Mesh Generation by IME
  - ✓ Scalability Evaluation by Adaptive Multilevel hCGA (AM-hCGA)
- AMG
  - ✓ Researches on AM-hCGA algorithm without ParMETIS
  - ✓ Researches on smoothers like preconditioned Chebyshev smoothers and MS-BMC-GS.
  - ✓ Communication optimization on halo communication using user-level RDMA on Flow type1.
- PinST
  - ✓ MGRIT Preconditioning convergence analysis
  - ✓ TSC method application to pHEAT-3D
  - ✓ PinST method to Navier-Stokes applications
  - ✓ PinST method for explicit method