機械学習に基づくマクロ経済変動の数理モデリング(jh200020-MDH)

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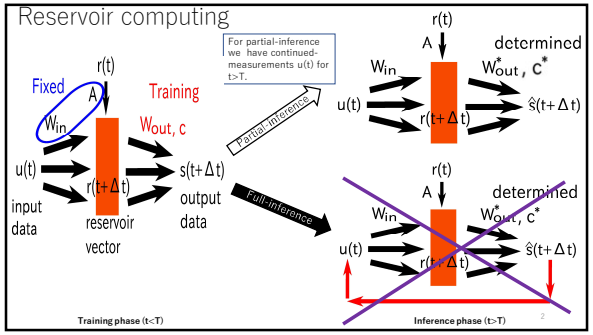
Reservoir computing



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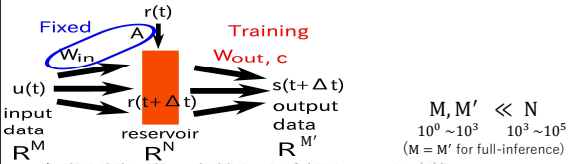
Reservoir computing



* $\mathbf{r}(t+\Delta t) = (1-\alpha)\mathbf{r}(t) + \alpha \tanh(\mathbf{A}\mathbf{r}(t) + \mathbf{W}_{in}\mathbf{u}(t))$
$\mathbf{A}, \mathbf{W}_{in}$: sparse random matrix, whose maximal eigenvalue is controlled
α: nonlinearity parameter ($\alpha = 0.3$).

We determine $\mathbf{W}_{out}$ and c s.t.

$$t^{\forall} < T \qquad \mathbf{W}_{out}\mathbf{r}(t+\Delta t) + c \approx \mathbf{s}(t+\Delta t).$$

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Reservoir computing: How to determine Wout and c ?

- Minimizing the quadratic form with respect to **Wout** and **c:**

$$\sum_{l=1}^{L} \|(\mathbf{W}_{\text{out}}\mathbf{r}(l\Delta t) + \mathbf{c}) - \mathbf{s}(l\Delta t)\|^2 + \beta[Tr(\mathbf{W}_{\text{out}}\mathbf{W}_{\text{out}}^T)]$$

regularization term to avoid overfitting

- Solution: $\hat{\mathbf{s}}(t) = \mathbf{W}_{\text{out}}^*\mathbf{r}(t) + c^*$ (Lukosevivsius and Jaeger, 2009)

$$\mathbf{W}_{\text{out}}^* = \delta\mathbf{S}\delta\mathbf{R}^T(\delta\mathbf{R}\delta\mathbf{R}^T + \beta\mathbf{I})^{-1}$$
$$c^* = -[\mathbf{W}_{\text{out}}^*\bar{\mathbf{r}} - \bar{\mathbf{s}}]$$

where $\bar{r} = \sum_{l=1}^{L}\mathbf{r}(l\Delta t)/\mathrm{L}$, $\bar{s} = \sum_{l=1}^{L}\mathbf{s}(l\Delta t)/L$, and $\mathbf{I}$ is the $N \times N$ identity matrix, $\delta\mathbf{R}$ (respectively, $\delta\mathbf{S}$) is the matrix whose l-th column is $\mathbf{r}(l\Delta t) - \bar{r}$ (respectively, $\mathbf{s}(l\Delta t) - \bar{s}$).

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Inference of macroscopic economic variables

We employ reservoir computing to infer current (time t) macroscopic economic variables *s* like GDP using economic data *u* available at time t.

- *u* : filtered economic data which can be observed at time t
  [stock price, interest rate, currency exchange rate]
- *s* : economic data which will be reported at time t+T
  (T: several months)
  [GDP, IIP, net export]

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Reference

1. Nakai, K. and Y. Saiki. (2018). Machine-learning inference of fluid variables from data using reservoir computing, Phys. Rev. E 98, 023111. https://arxiv.org/abs/1805.09917

2. Nakai, K. and Saiki, Y. (2020). Machine-learning construction of a model for a macroscopic fluid variable using the delay-coordinate of a scalar observable, , Discrete and Continuous Dynamical Systems S, Online first. https://arxiv.org/pdf/1903.05770

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