[EX24606]

情報量に偏りや欠損がある時系列データを用いた機械学習時間発展モデリング

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1 Introduction.

Reservoir computing, a brain-inspired machine-learning technique that employs a data-driven dynamical system, is effective in predicting time series and frequency spectra in chaotic behaviors, including fluid flow and global atmospheric dynamics [1, 2, 4, 5, 6].

We show that the effect of training data for reservoir computing on the reconstruction of chaotic dynamics. Our findings indicate that a training time series comprising a few periodic orbits of low periods can sufficiently reconstruct the chaotic attractor. We also demonstrate that biased training data do not negatively impact reconstruction success. Our method's ability to reconstruct a physical measure is considerably better than the so-called cycle expansion approach, which relies on weighted averaging. In this study, using periodic orbits to generate biased small training data is significant to understanding how training data affect the construct data-driven model.

2 Reservoir computation.

What's Reservoir computation?

- a relatively high-dimentional fixed neural-network composed of simple nonlinear dynamical systems
- determination of output layer
- For Lorenz system and Kuramoto–Sivashinsky system, inference [1, 5, 6]

2.1 Procedure of training.

○ **1st step (generating a reservoir vector)**

$$\mathbf{r}(t+\Delta t) = (1-\alpha)\mathbf{r}(t) + \alpha \tanh(\mathbf{A}\mathbf{r}(t) + \mathbf{W}_{\mathrm{in}}\mathbf{u}(t) + \xi\mathbf{1})$$

$\mathbf{A}, \mathbf{W}_{\mathrm{in}}$: sparse random matrix, whose maximal eigenvalue is controlled.

ξ: a scalar constant

$\mathbf{1} = (1, 1, , , , , 1)^T$

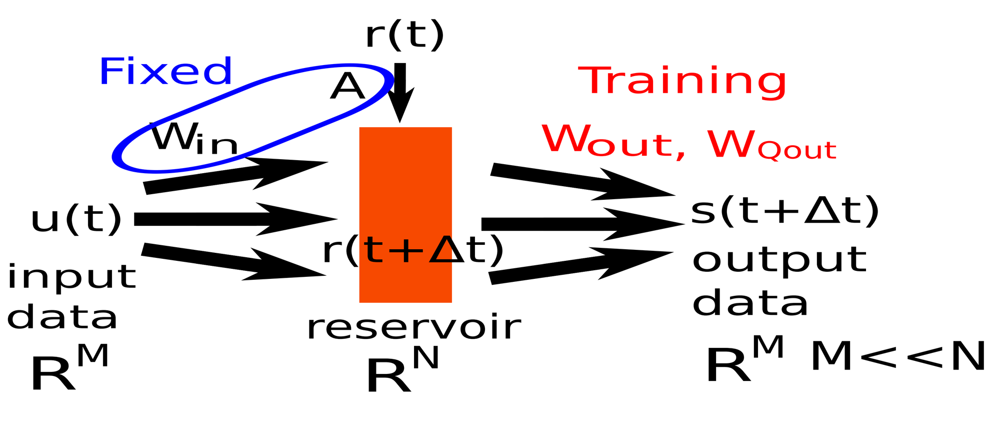


Figure 1: Schematic picture of a reservoir computing (training phase)

○ **2nd step (determination of output layer)**

We determine $\mathbf{W}_{\mathrm{out}}$ and $\mathbf{W}^*_{\mathrm{Qout}}$ s.t.

$$^\forall t < T \qquad \mathbf{W}_{\mathrm{out}}\mathbf{r}(t+\Delta t) + \mathbf{r}(t)^{\mathrm{T}}\mathbf{W}^*_{\mathrm{Qout}}\mathbf{r}(t) \approx \mathbf{s}(t+\Delta t).$$

$\mathbf{W}^*_{\mathrm{out}} \in \mathbb{R}^{M\times N}$ is a matrix.

$\mathbf{W}^*_{\mathrm{Qout}} \in \mathbb{R}^{M\times N\times N}$ is a tensor [7].

2.2 Procedure of inference.

Using the $\mathbf{W}^*_{\mathrm{out}}$ and $\mathbf{W}^*_{\mathrm{Qout}}$, we infer the time-series $\mathbf{s}$.

$$\hat{\mathbf{s}}(t) = \mathbf{W}^*_{\mathrm{out}}\mathbf{r}(t) + \mathbf{r}(t)^{\mathrm{T}}\mathbf{W}^*_{\mathrm{Qout}}\mathbf{r}(t)$$

$$\mathbf{r}(t+\Delta t) = (1-\alpha)\mathbf{r}(t) + \alpha \tanh(\mathbf{A}\mathbf{r}(t) + \mathbf{W}_{\mathrm{in}}\hat{\mathbf{s}}(t) + \xi\mathbf{1}).$$

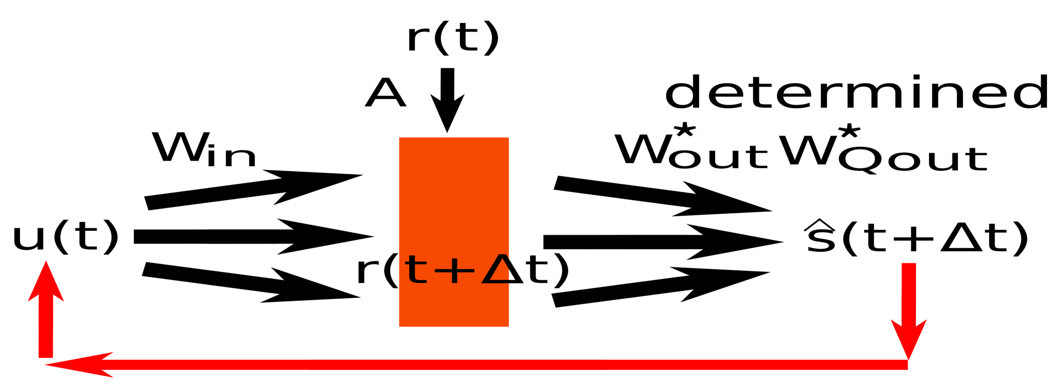
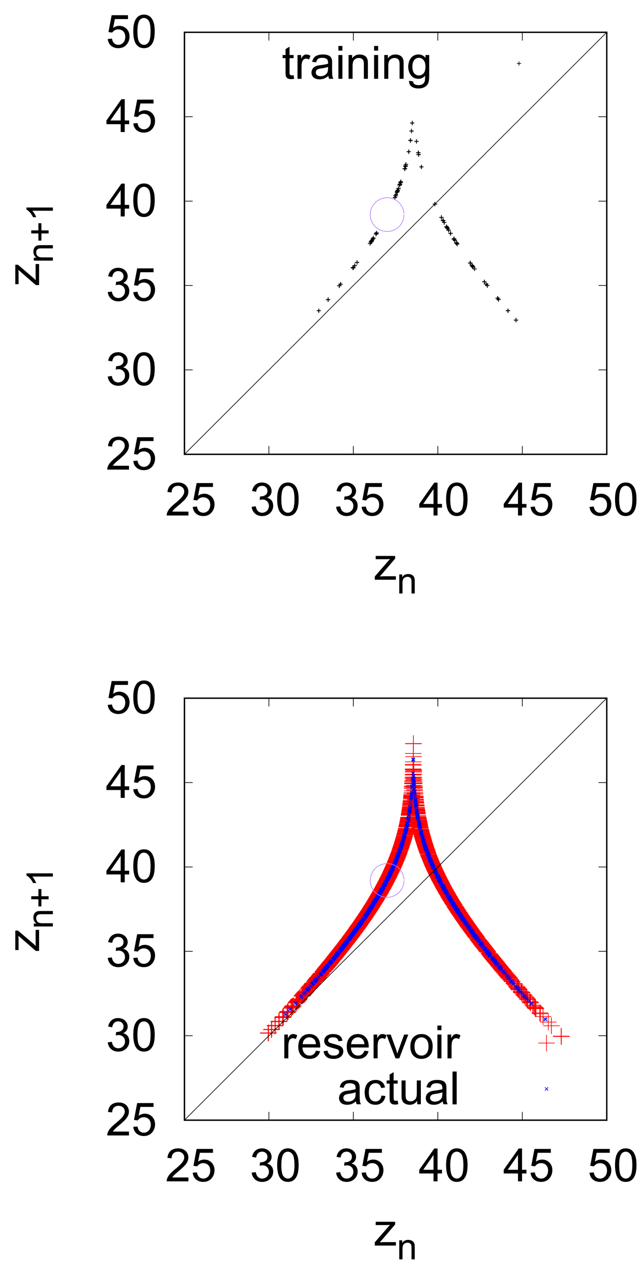


Figure 2: Schematic picture of a reservoir computing (prediction phase)

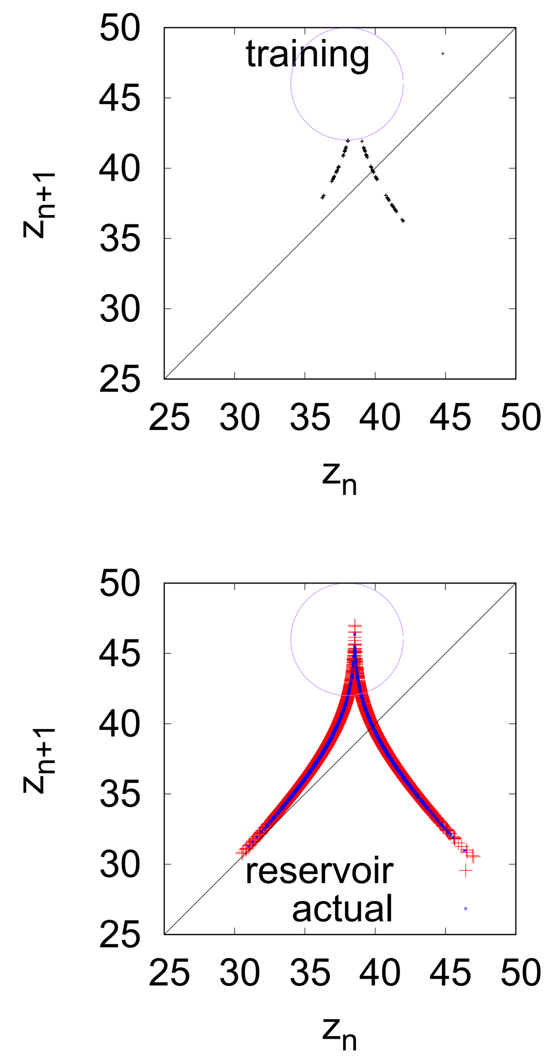
This reservoir system corresponds to the data-driven model of $\mathbf{u}$.

3 Results

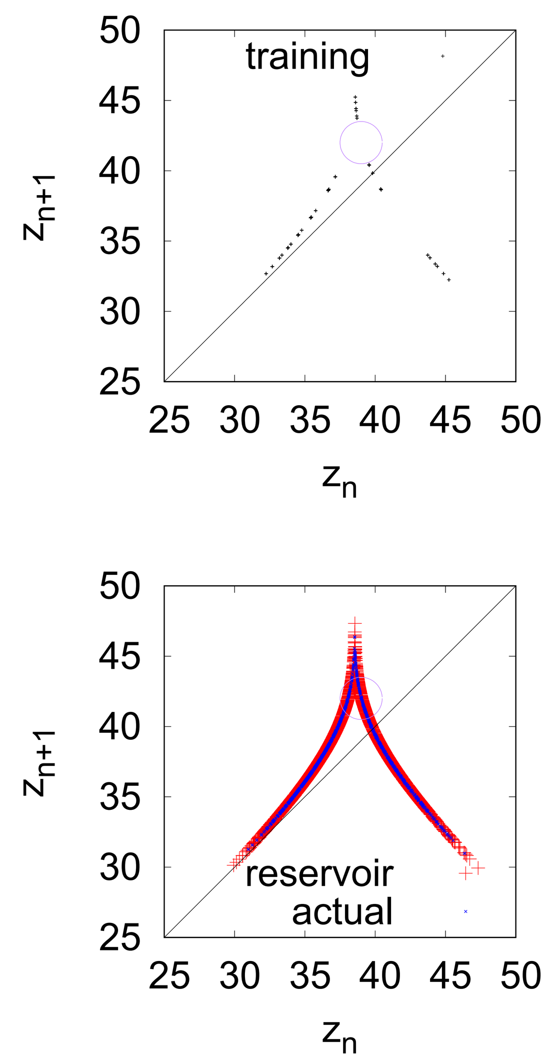
We investigate the effect of biased training data on modeling.



The top panel shows training data in the return plots defined by the maximal value of the z variable, and the bottom panel shows the corresponding return plots created from the reservoir model. Even if some periodic orbits that pass through a certain region surrounded by the circle are excluded from the training dataset, the reservoir model reconstructs the attractor. The regions are surrounded by the circles: $B_1(37, 39.2)$.



Excluded from periodic orbits which pass through the region surrounded by the circle $B_4(38, 46)$.



Excluded from periodic orbits which pass through the region surrounded by the circle $B_{1.5}(39, 42)$.

Reference

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