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大規模計算機のための高次精度時空間計算手法による非定常流体シミュレーション

Introduction

It is challenging to compute unsteady flow problems that include interaction between different time-scale phenomena in a reasonable amount of computing time. One of the reasons for the difficulty in computing unsteady problems in a reasonable amount of time is that typically parallel computations are limited by the number of unknowns we can solve simultaneously.

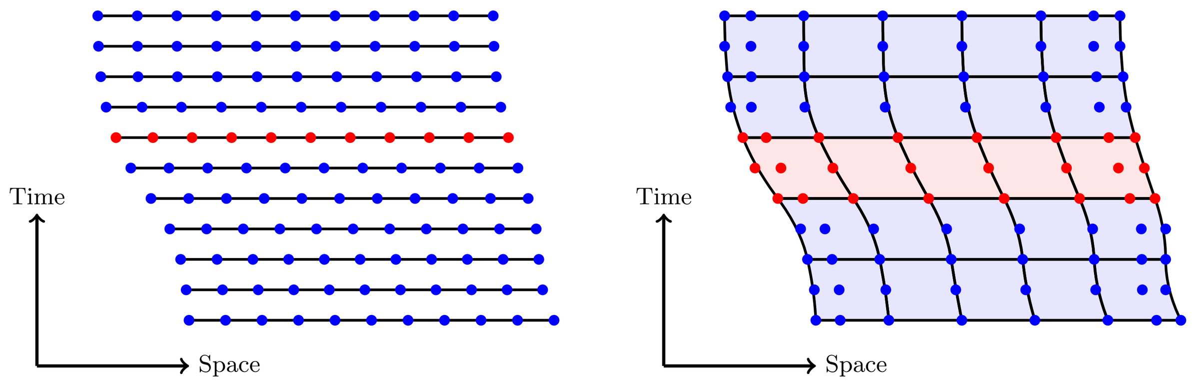

Figure: Unknowns to be parallelized in unsteady flow computation. Conventional (*left*), space–time (*right*)

To shorten the computing time, we need to reduce the number of time steps by increasing the time-step size. The only way to maintain the spatial resolution while increasing the grid spacing is to use higher-order functions.
We provide a stability and accuracy analysis. Linear functions and C^1 B-splines in space are potentially fourth-order and sixth-order accurate, respectively. But, we quickly lose the higher-order accuracy at high Courant–Friedrichs–Lewy (CFL) numbers, unless we also use higher-order basis functions in time.

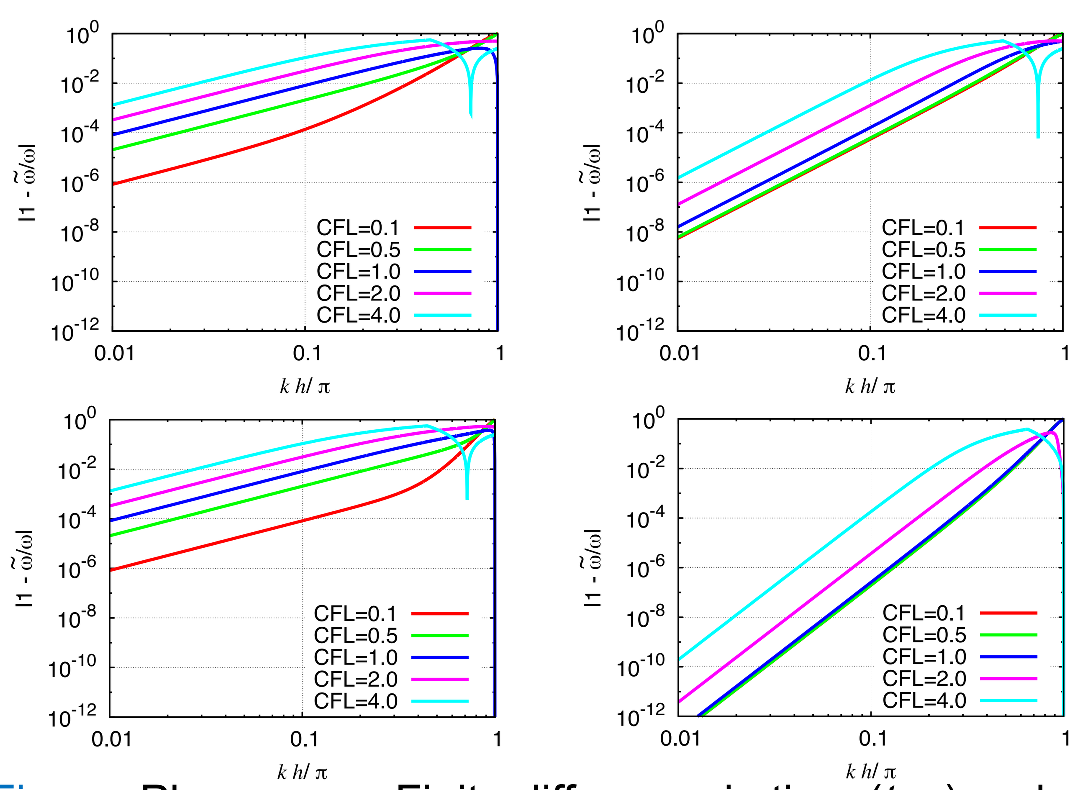

Figure: Phase error. Finite difference in time (*top*) and quadratic B-splines in time (*bottom*). Linear in space (*left*) and C^1 B-splines in space (*right*)

Space–Time Isogeometric Analysis

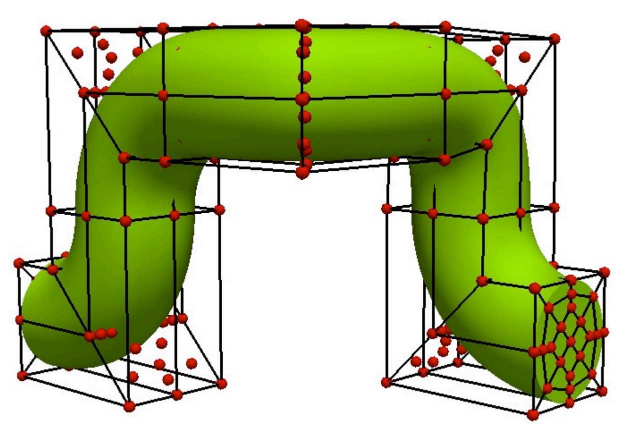

Figure: 3D pipe mesh with NURBS

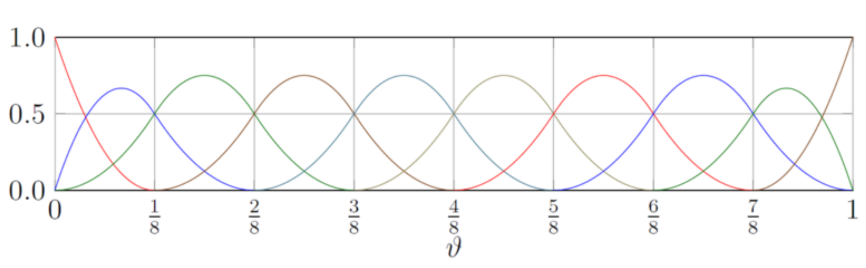

Figure: NURBS basis functions

Isogeometric analysis (IGA) was introduced in 2005 [1]. The method was motivated to have a tight integration in computer-aided design (CAD) modeling and computer-aided engineering (CAE). Both CAD and IGA use non-uniform rational B-spline (NURBS) as basis functions. NURBS and space–time isogeometric analysis (ST-IGA) possess useful mathematical properties:

- Precise and efficient representation of geometry and motion
- Mesh refinement without changing the geometry
- High continuity between elements
- High accuracy and good stability

Mesh Generation

Though the IGA is known to be powerful in computational analysis, the difficulty of the mesh generation makes its application to flow problems not so easy. To make the IGA more practical in computational flow analysis with complex geometries, NURBS volume mesh generation needs to be easier and more automated. We are developing a general-purpose NURBS mesh generation method [2], that enables us to use the ST methods and isogeometric discretization with a mesh generation burden that is comparable to what one typically faces in conventional methods.
Basic concepts:

- Generate a multi-block structured finite element (FE) mesh
- Split into blocks
- Projection to determine the NURBS control points

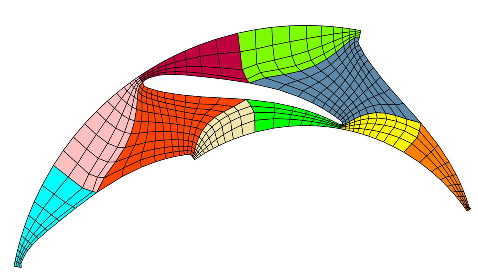

Figure: FE mesh blocks

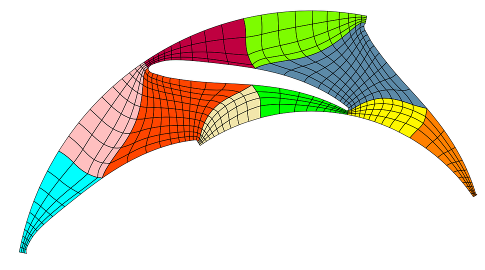

Figure: NURBS patches

We see each block as a precursor to a NURBS patch. The method is expected to retain the refinement distribution and element quality of the multi-block structured mesh that we start with. Once we generate the NURBS mesh, the computational efficiency is substantially increased.

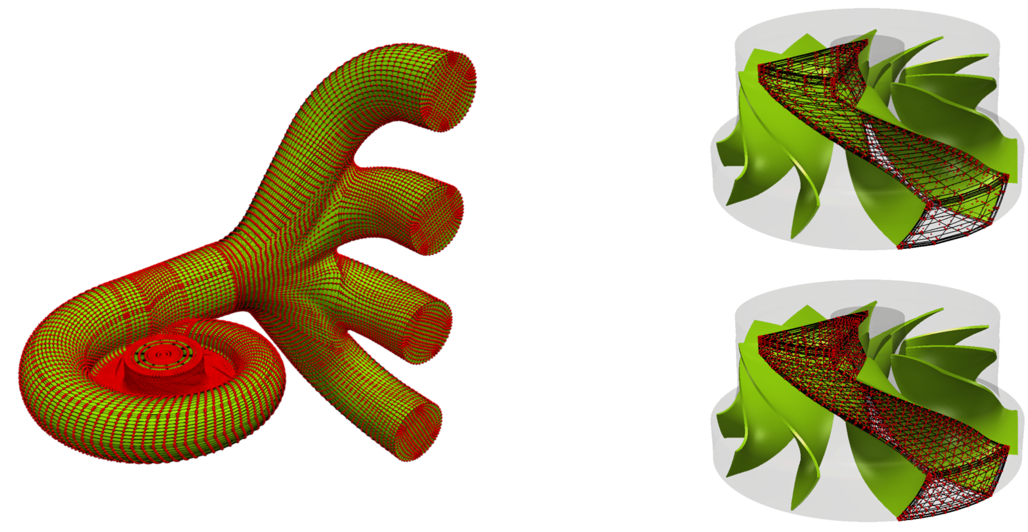

Figure: Control mesh of the turbocharger

Stabilization Parameters

We are now successfully computing many challenging engineering problems with the ST methods and isogeometric discretization. At the same time, in more complex cases, we are addressing additional numerical challenges related to the local length scale definition for isogeometric discretization. That motivated the research on a well-reasoned method for determining the local length scales.
We introduced the new local length definition in [3].

Original Form

$$h_{\mathrm{RGN}} = 2\left(\sum_{\alpha=1}^{n_{\mathrm{ent}}}\sum_{a=1}^{n_{\mathrm{ens}}}\left|\mathbf{r}\cdot\nabla N_a^{\alpha}\right|\right)^{-1}$$

New Form

$$Q_{ij} = \frac{\partial x_i}{\partial \xi_j}$$

$$\hat{\mathbf{Q}} = \mathbf{Q}\mathbf{D}^{-1}$$

$$\mathbf{G} = \hat{\mathbf{Q}}^{-T}\hat{\mathbf{Q}}^{-1}$$

$$h_{\mathrm{RQD}} = 2\left(\mathbf{rr} : \mathbf{G}\right)^{-\frac{1}{2}}$$

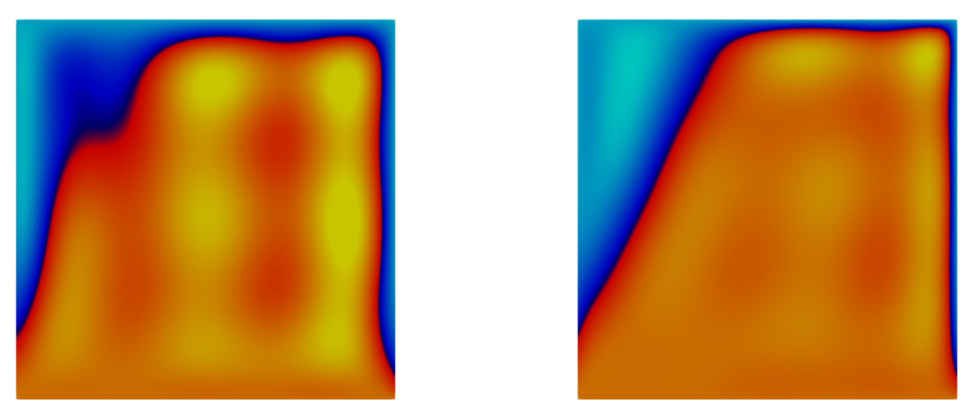

Figure: Advection skew to the mesh. Mesh: 1×1. Stabilization parameter: RQD-**I** (*left*) and RQD-*p***I** (*right*)

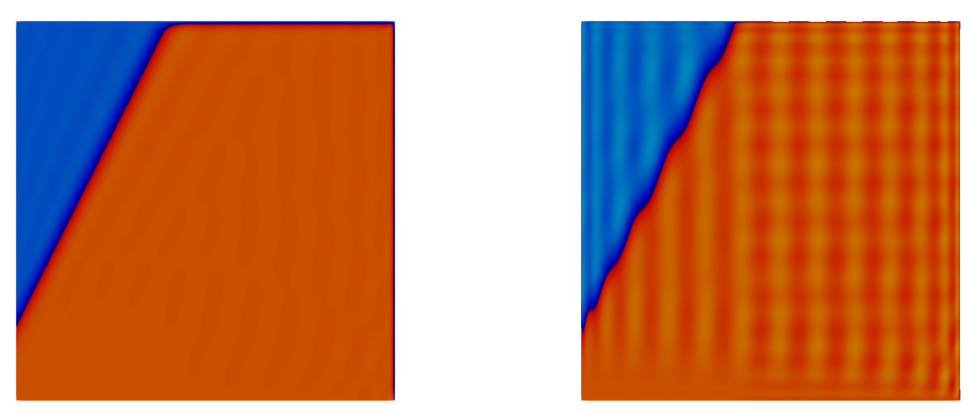

Figure: Advection skew to the mesh. Mesh: 20×20. Stabilization parameter: RQD-**I** (*left*) and RQD-*p***I** (*right*)

Computations in Engineering Problem

We focus on turbocharger computational flow analysis with a method that possesses higher accuracy in spatial and temporal representations [4]. The core computational methods:

- ST Variational Multiscale (ST-VMS) method, which is a stabilized formulation that also serves as a turbulence model
- ST-IGA provides accurate geometry representation and increased solution accuracy
- ST Slip Interface (ST-SI) method, which maintains high-resolution representation of the boundary layers
- New stabilization parameters and element lengths are used in the ST-VMS and ST-SI

We compute the flow for a full intake/exhaust cycle, which is much longer than the turbine rotation cycle because of higher turbine speeds, and the long duration required is an additional computational challenges.

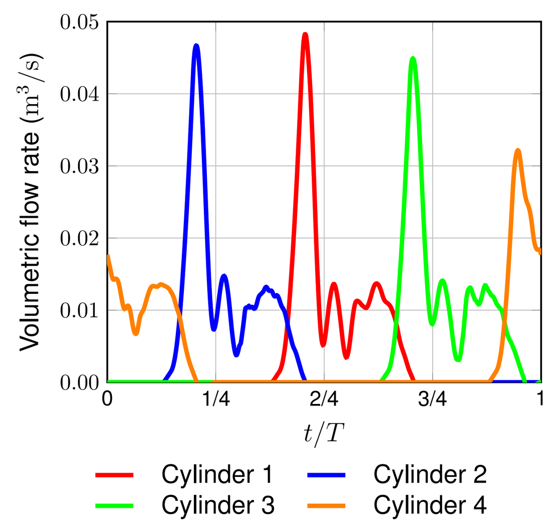

Figure: Inflow profiles

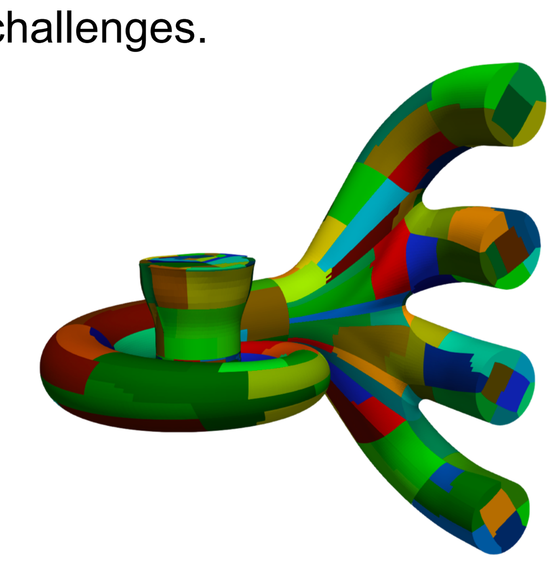

Figure: Partitioned mesh

The time-step size based on 90 steps per cycle is very small compare to typical computations of rotating machineries. With that large time-step size overall CFL at the peak flow rate is kept around one. This means that the mesh is also coarse without loosing the geometry and solution accuracy, which is archived by the power of the isogeometric discretization.

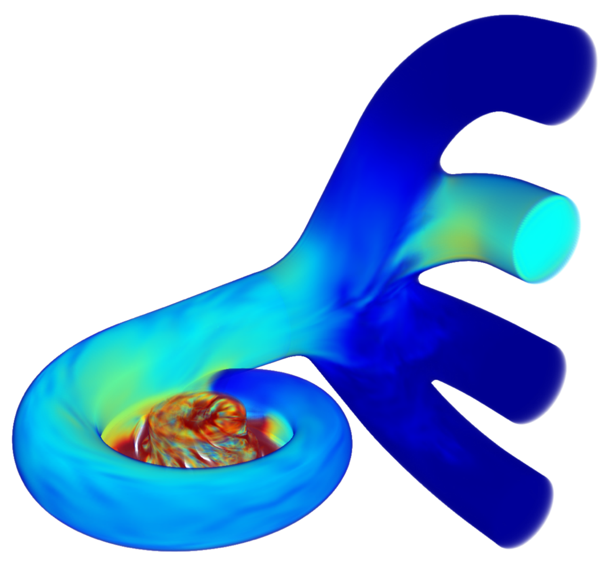

Figure: Velocity magnitude

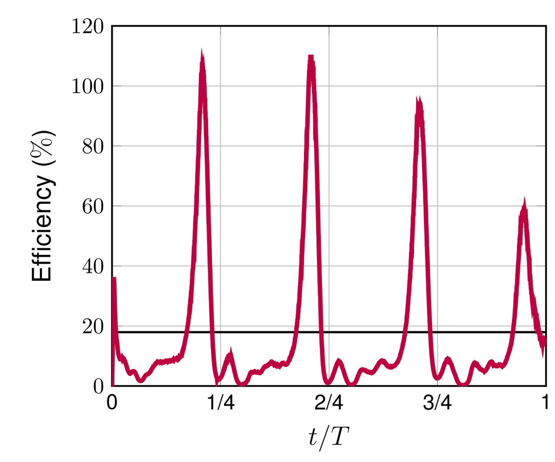

Figure: Turbine efficiency

Now, we are trying the computations with higher-order temporal basis functions.

Table: Number of unknowns

Temporal basis functions	Number of unknowns	Number of time steps
Finite difference in time	1,743,460	27,000
Space–Time (linear in time)	3,486,920	2,700
Space–Time (cubic in time)	5,230,380	900

References

[1] T.J.R. Hughes, J.A. Cottrell, and Y. Bazilevs, "Isogeometric analysis: CAD, finite elements, NURBS, exact geometry, and mesh refinement", *Computer Methods in Applied Mechanics and Engineering*, **194** (2005) 4135–4195.
[2] Y. Otoguro, K. Takizawa and T.E. Tezduyar, "Space–time VMS computational flow analysis with isogeometric discretization and general-purpose NURBS mesh generation method", *Computers & Fluids*, **158** (2017) 189–200, DOI: 10.1016/j.compfluid.2017.04.017.
[3] K. Takizawa, T.E. Tezduyar and Y. Otoguro "Stabilization and discontinuity-capturing parameters for space–time flow computations with finite element and isogeometric discretizations", *Computational Mechanics,* (2018), published online. DOI: 10.1007/s00466-018-1557-x.
[4] Y. Otoguro, K. Takizawa, T.E. Tezduyar, K. Nagaoka and S. Mei, "Turbocharger turbine and exhaust manifold flow computation with the Space–Time Variational Multiscale Method and Isogeometric Analysis", *Computers & Fluids,* (2018), published online. DOI: 10.1016/j.compfluid.2018.05.019